$$M=\left\{\begin{bmatrix}0 & a & b\\ -a & 0 & c\\ -b & -c & 0\end{bmatrix} : a,b,c\in\mathbb{R}\right\}$$

$$M=\left[\left\{\begin{bmatrix}0 & 1 & 0\\ -1 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}, \begin{bmatrix}0 & 0 & 1\\ 0 & 0 & 0\\ -1 & 0 & 0\end{bmatrix}, \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 1\\ 0 & -1 & 0\end{bmatrix}\right\}\right]$$

$$N=\left\{\begin{bmatrix}0 & 0 & 0\\ x & 0 & 0\\ x & x & 0\end{bmatrix} : x\in\mathbb{R}\right\}=\left[\left\{\begin{bmatrix}0 & 0 & 0\\ 1 & 0 & 0\\ 1 & 1 & 0\end{bmatrix}\right\}\right]$$

$$M+N=\left[\left\{\overbrace{\begin{bmatrix}0 & 1 & 0\\ -1 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}}^{A_1}, \overbrace{\begin{bmatrix}0 & 0 & 1\\ 0 & 0 & 0\\ -1 & 0 & 0\end{bmatrix}}^{A_2}, \overbrace{\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 1\\ 0 & -1 & 0\end{bmatrix}}^{A_3},\right.\right.$$

$$\left.\left.\underbrace{\begin{bmatrix}0 & 0 & 0\\ 1 & 0 & 0\\ 1 & 1 & 0\end{bmatrix}}_{A_4}\right\}\right]$$

Ortonormirajmo bazu $\{A_1, A_2, A_3, A_4\}$

G-S postupkom.

$$\bullet\quad E_1 = \frac{A_1}{\|A_1\|} = \frac{A_1}{\sqrt{2}} = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$B_2 = A_2 - \underbrace{\langle A_2 | E_1 \rangle}_{0} E_1 = A_2$$

$$\bullet\quad E_2 = \frac{B_2}{\|B_2\|} = \frac{A_2}{\|A_2\|} = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$B_3 = A_3 - \underbrace{\langle A_3 | E_1 \rangle}_{0} E_1 - \underbrace{\langle A_3 | E_2 \rangle}_{0} E_2 = A_3$$

$$\bullet\quad E_3 = \frac{B_3}{\|B_3\|} = \frac{A_3}{\|A_3\|} = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$B_4 = A_4 - \langle A_4 | E_1 \rangle E_1 - \langle A_4 | E_2 \rangle E_2 - \langle A_4 | E_3 \rangle E_3$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

- $E_4 = \dfrac{B_4}{\|B_4\|} = \dfrac{1}{\sqrt{6}}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

$$A = \begin{bmatrix} 4 & 1 & -2 \\ 5 & 3 & 2 \\ 4 & 2 & 0 \end{bmatrix}$$

Najbolja aproksimacija matrice A matricama iz potprostora M+N je ortogonalna projekcija matrice A na M+N, a ona je dana s:

$$P(A) = \langle A|E_1\rangle E_1 + \langle A|E_2\rangle E_2 + \langle A|E_3\rangle E_3 + \langle A|E_4\rangle E_4$$

$$= \frac{1-5}{2}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{-2-4}{2}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$+ \frac{2-2}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} + \frac{1\cancel{-2}+5\cancel{+2}+4+2}{6}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -2 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & -3 \\ 0 & 0 & 0 \\ 3 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 2 & 2 \\ 2 & 0 & 2 \\ 2 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ 4 & 0 & 2 \\ 5 & 2 & 0 \end{bmatrix}$$