6. zad, poglavje 3.3.

$$f_{v,w}(A) = \langle Av | w \rangle$$

$$v = (1,2), \quad w = (3,1)$$

$$f_{v,w}(A) = \langle A(1,2) | (3,1) \rangle$$

$$A(1,2) = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} a+2b \\ c+2d \end{bmatrix}$$

$$\sim (a+2b, c+2d)$$

$$f_{v,w}(A) = \langle (a+2b, c+2d) | (3,1) \rangle$$

$$= 3(a+2b) + (c+2d)$$

$$= 3a + 6b + c + 2d \qquad (*)$$

$$f_{v,w}(A) = tr(AB^T) = \langle A | B \rangle \quad \forall A$$

$$B = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$$

$$f_{v,w}\left(\begin{bmatrix}1&0\\0&0\end{bmatrix}\right)=\left\langle\begin{bmatrix}1&0\\0&0\end{bmatrix}\middle|\begin{bmatrix}x&y\\z&w\end{bmatrix}\right\rangle=x$$

$$f_{v,w}\left(\begin{bmatrix}0&1\\0&0\end{bmatrix}\right)=\left\langle\begin{bmatrix}0&1\\0&0\end{bmatrix}\middle|\begin{bmatrix}x&y\\z&w\end{bmatrix}\right\rangle=y$$

$$f_{v,w}\left(\begin{bmatrix}0&0\\1&0\end{bmatrix}\right)=\left\langle\begin{bmatrix}0&0\\1&0\end{bmatrix}\middle|\begin{bmatrix}x&y\\z&w\end{bmatrix}\right\rangle=z$$

$$f_{v,w}\left(\begin{bmatrix}0&0\\0&1\end{bmatrix}\right)=\left\langle\begin{bmatrix}0&0\\0&1\end{bmatrix}\middle|\begin{bmatrix}x&y\\z&w\end{bmatrix}\right\rangle=w$$

S druge strane, iz $(*)$

$$\Rightarrow \quad f_{v,w}\left(\begin{bmatrix}1&0\\0&0\end{bmatrix}\right)=3 \quad\Rightarrow\quad x=3$$

$$f_{v,w}\left(\begin{bmatrix}0&1\\0&0\end{bmatrix}\right)=6 \quad\Rightarrow\quad y=6$$

$$f_{v,w}\left(\begin{bmatrix}0&0\\1&0\end{bmatrix}\right)=1 \quad\Rightarrow\quad z=1$$

$$f_{v,w}\left(\begin{bmatrix}0&0\\0&1\end{bmatrix}\right)=2 \quad\Rightarrow\quad w=2$$

$$\Longrightarrow B=\begin{bmatrix}3&6\\1&2\end{bmatrix}$$

7. zad, poglavlje 3.3.

$$\langle p|q\rangle = p(-1)q(-1) + p(0)q(0) + p'(0)q'(0)$$

$$f : \mathcal{P}_2 \to \mathbb{R}$$

$$f(p) = \langle p|p_1\rangle + \langle p|p_2\rangle + p(0)$$

$$p_1(t) = 2, \quad p_2(t) = t$$

$$q(t) = at^2 + bt + c$$

$$f(p) = \langle p|q\rangle \quad \forall\, p \in \mathcal{P}_2$$

$$e_1(t) = 1, \quad e_2(t) = t, \quad e_3(t) = t^2$$

$$f(e_1) = \langle e_1|p_1\rangle + \langle e_1|p_2\rangle + e_1(0)$$
$$= (1\cdot 2 + 1\cdot 2 + 0\cdot 0) + (1\cdot(-1) + 1\cdot 0 + 0\cdot 1)$$
$$+ 1 = 4 - 1 + 1 = 4$$

$$f(e_2) = \langle e_2|p_1\rangle + \langle e_2|p_2\rangle + e_2(0) =$$
$$= ((-1)\cdot 2 + 0\cdot 2 + 1\cdot 0) + ((-1)^2 + 0^2 + 1^2) + 0 = 0$$

$$f(e_3) = \langle e_3 | P_1 \rangle + \langle e_3 | P_2 \rangle + e_3(0)$$

$$= (1\cdot 2 + 0\cdot 2 + 0\cdot 0) + (1\cdot(-1) + 0\cdot 0 + 0\cdot 1)$$

$$+ 0 = 2 - 1 = 1$$

$$4 = f(e_1) = \langle e_1 | at^2 + bt + c \rangle$$

$$= e_1(-1)\cdot(a-b+c) + e_1(0)\cdot c$$

$$+ e_1'(0)\cdot(b)$$

$$= a-b+c+c = a-b+2c$$

$$0 = f(e_2) = \langle e_2 | at^2 + bt + c \rangle$$

$$= e_2(-1)\cdot(a-b+c) + e_2(0)\cdot c$$

$$+ e_2'(0)\cdot b = -a+b-c+b$$

$$= -a+2b-c$$

$$1 = f(e_3) = \langle e_3 | at^2 + bt + c \rangle =$$

$$= e_3(-1)(a-b+c) + e_3(0)\cdot c + e_3'(0)\cdot b$$

$$= a-b+c$$

$$\Rightarrow \begin{aligned} a-b+2c&=4\\ -a+2b-c&=0\\ a-b+c&=1 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 4\\ -1 & 2 & -1 & 0\\ 1 & -1 & 1 & 1 \end{array}\right] \begin{array}{l} \\ \\ /\cdot(-1) \end{array} \oplus$$

$$\sim \left[\begin{array}{ccc|c} 0 & 0 & 1 & 3\\ -1 & 2 & -1 & 0\\ 1 & -1 & 1 & 1 \end{array}\right] \oplus \left[\begin{array}{ccc|c} 0 & 0 & 1 & 3\\ 0 & 1 & 0 & 1\\ 1 & -1 & 1 & 1 \end{array}\right] \begin{array}{l} /\cdot(-1) \\ \oplus \quad \oplus \\ \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 0 & 0 & 1 & 3\\ 0 & 1 & 0 & 1\\ 1 & 0 & 0 & -1 \end{array}\right] \Rightarrow \begin{aligned} c&=3\\ b&=1\\ a&=-1 \end{aligned}$$

$$\Rightarrow \boxed{q(t) = -t^2 + t + 3}$$