

BOLTZMANNOVA RASPODJELA

DODATNI ZADACI - RJEŠENJA

10. Izračunajte relativnu populacijsku razliku ($\Delta N/N_0$) za protone u polju od a) 0,30 T b) 1,5 T i c) 10 T pri 25°C ($\gamma = 2,6752 \cdot 10^8 \text{ rad T}^{-1} \text{ s}^{-1}$).

$$\vartheta = 25^\circ\text{C} \Rightarrow T = 298,15 \text{ K} \quad \gamma(^1\text{H}) = 2,6752 \cdot 10^8 \text{ rad T}^{-1} \text{ s}^{-1}$$

a) $B_0 = 0,30 \text{ T}$

$$\nu_L = \frac{\gamma B_0}{2\pi} = \frac{2,6752 \cdot 10^8 \text{ rad T}^{-1} \text{ s}^{-1} \cdot 0,30 \text{ T}}{2\pi} = 12,77 \cdot 10^6 \text{ Hz}$$

$$\Delta E = h \nu_L = 6,626 \cdot 10^{-34} \text{ J s} \cdot 12,77 \cdot 10^6 \text{ Hz} = 8,463 \cdot 10^{-27} \text{ J}$$

$$\frac{\Delta N}{N_0} = 1 - e^{-\frac{\Delta E}{kT}} = 2,057 \cdot 10^{-6}$$

b) $B_0 = 1,5 \text{ T}$

$$\nu_L = \frac{\gamma B_0}{2\pi} = \frac{2,6752 \cdot 10^8 \text{ rad T}^{-1} \text{ s}^{-1} \cdot 1,5 \text{ T}}{2\pi} = 63,87 \cdot 10^6 \text{ Hz}$$

$$\Delta E = h \nu_L = 6,626 \cdot 10^{-34} \text{ J s} \cdot 63,87 \cdot 10^6 \text{ Hz} = 4,232 \cdot 10^{-26} \text{ J}$$

$$\frac{\Delta N}{N_0} = 1 - e^{-\frac{\Delta E}{kT}} = 1,028 \cdot 10^{-5}$$

c) $B_0 = 10 \text{ T}$

$$\nu_L = \frac{\gamma B_0}{2\pi} = \frac{2,6752 \cdot 10^8 \text{ rad T}^{-1} \text{ s}^{-1} \cdot 10 \text{ T}}{2\pi} = 425,77 \cdot 10^6 \text{ Hz}$$

$$\Delta E = h \nu_L = 6,626 \cdot 10^{-34} \text{ J s} \cdot 425,77 \cdot 10^6 \text{ Hz} = 2,821 \cdot 10^{-25} \text{ J}$$

$$\frac{\Delta N}{N_0} = 1 - e^{-\frac{\Delta E}{kT}} = 6,856 \cdot 10^{-5}$$

11. Izračunajte relativnu populacijsku razliku ($\Delta N/N_0$) za jezgre ^{13}C u polju od a) 0,30 T b) 1,5 T i c) 10 T pri 25°C? ($\gamma = 6,7283 \cdot 10^7 \text{ rad T}^{-1} \text{ s}^{-1}$).

$$\vartheta = 25^\circ\text{C} \Rightarrow T = 298,15 \text{ K} \quad \gamma(^{13}\text{C}) = 6,7283 \cdot 10^7 \text{ rad T}^{-1} \text{ s}^{-1}$$

a) $B_0 = 0,30 \text{ T}$

$$\nu_L = \frac{\gamma B_0}{2\pi} = \frac{6,7283 \cdot 10^7 \text{ rad T}^{-1} \text{ s}^{-1} \cdot 0,30 \text{ T}}{2\pi} = 3,21 \cdot 10^6 \text{ Hz}$$

$$\Delta E = h \nu_L = 6,626 \cdot 10^{-34} \text{ J s} \cdot 3,21 \cdot 10^6 \text{ Hz} = 2,129 \cdot 10^{-27} \text{ J}$$

$$\frac{\Delta N}{N_0} = 1 - e^{-\frac{\Delta E}{kT}} = 5,173 \cdot 10^{-7}$$

b) $B_0 = 1,5 \text{ T}$

$$\nu_L = \frac{\gamma B_0}{2\pi} = \frac{6,7283 \cdot 10^7 \text{ rad T}^{-1} \text{ s}^{-1} \cdot 1,5 \text{ T}}{2\pi} = 16,06 \cdot 10^6 \text{ Hz}$$

$$\Delta E = h \nu_L = 6,626 \cdot 10^{-34} \text{ J s} \cdot 16,06 \cdot 10^6 \text{ Hz} = 1,064 \cdot 10^{-26} \text{ J}$$

$$\frac{\Delta N}{N_0} = 1 - e^{-\frac{\Delta E}{kT}} = 2,587 \cdot 10^{-6}$$

c) $B_0 = 10 \text{ T}$

$$\nu_L = \frac{\gamma B_0}{2\pi} = \frac{6,7283 \cdot 10^7 \text{ rad T}^{-1} \text{ s}^{-1} \cdot 10 \text{ T}}{2\pi} = 107,08 \cdot 10^6 \text{ Hz}$$

$$\Delta E = h \nu_L = 6,626 \cdot 10^{-34} \text{ J s} \cdot 107,08 \cdot 10^6 \text{ Hz} = 7,095 \cdot 10^{-26} \text{ J}$$

$$\frac{\Delta N}{N_0} = 1 - e^{-\frac{\Delta E}{kT}} = 1,724 \cdot 10^{-5}$$

12. Izračunajte omjer broja molekula Cl_2 u osnovnom i prvom pobuđenom vibracijskom stanju pri a) 298 K i b) 500 K. Valni broj vibracije molekule Cl_2 iznosi $559,7 \text{ cm}^{-1}$.

$$\tilde{\nu}(\text{Cl}_2) = 559,7 \text{ cm}^{-1} = 559,7 \cdot 10^2 \text{ m}^{-1}$$

$$\Delta E = h c \tilde{\nu} = 6,626 \cdot 10^{-34} \text{ J s} \cdot 3 \cdot 10^8 \text{ m s}^{-1} \cdot 559,7 \cdot 10^2 \text{ m}^{-1} = 1,113 \cdot 10^{-20} \text{ J}$$

a) $T = 298 \text{ K}$

$$\frac{N_j}{N_o} = e^{-\frac{\Delta E}{kT}} = 0,067$$

b) $T = 500 \text{ K}$

$$\frac{N_j}{N_o} = e^{-\frac{\Delta E}{kT}} = 0,199$$

13. Izračunajte omjer broja molekula Br_2 u osnovnom i prvom pobuđenom vibracijskom stanju pri a) 298 K i b) 800 K. Valni broj vibracije molekule Br_2 iznosi $321,0 \text{ cm}^{-1}$.

$$\tilde{\nu}(\text{Br}_2) = 321,0 \text{ cm}^{-1} = 321,0 \cdot 10^2 \text{ m}^{-1}$$

$$\Delta E = h c \tilde{\nu} = 6,626 \cdot 10^{-34} \text{ J s} \cdot 3 \cdot 10^8 \text{ m s}^{-1} \cdot 321,0 \cdot 10^2 \text{ m}^{-1} = 6,381 \cdot 10^{-21} \text{ J}$$

a) $T = 298 \text{ K}$

$$\frac{N_j}{N_o} = e^{-\frac{\Delta E}{kT}} = 0,212$$

b) $T = 800 \text{ K}$

$$\frac{N_j}{N_o} = e^{-\frac{\Delta E}{kT}} = 0,561$$

14. Za Na^+ (589,3 nm) i Mg^{2+} (457,1 nm) ione usporedite omjer broja iona u 3p pobuđenom stanju u odnosu na osnovno pri a) 2100 K; b) 2900 K i c) u induktivno spregnutoj plazmi pri 6000 K.

$$\tilde{\nu}(\text{Na}^+) = 589,3 \text{ nm} = 589,3 \cdot 10^{-9} \text{ m} \qquad 3s \Rightarrow g_o = 2$$

$$\tilde{\nu}(\text{Mg}^{2+}) = 457,1 \text{ nm} = 457,1 \cdot 10^{-9} \text{ m} \qquad 3p \Rightarrow g_j = 6$$

$$\Delta E(\text{Na}^+) = \frac{hc}{\lambda(\text{Na}^+)} = \frac{6,626 \cdot 10^{-34} \text{ J s} \cdot 3 \cdot 10^8 \text{ m s}^{-1}}{589,3 \cdot 10^{-9} \text{ m}} = 3,373 \cdot 10^{-19} \text{ J}$$

$$\Delta E(\text{Mg}^{2+}) = \frac{hc}{\lambda(\text{Mg}^{2+})} = \frac{6,626 \cdot 10^{-34} \text{ J s} \cdot 3 \cdot 10^8 \text{ m s}^{-1}}{457,1 \cdot 10^{-9} \text{ m}} = 4,349 \cdot 10^{-19} \text{ J}$$

a) $T = 2100 \text{ K}$

$$\mathbf{Na}^+ \quad \frac{N_j}{N_o} = \frac{g_j}{g_o} e^{-\frac{\Delta E}{kT}} = 2,643 \cdot 10^{-5} \qquad \mathbf{Mg}^{2+} \quad \frac{N_j}{N_o} = \frac{g_j}{g_o} e^{-\frac{\Delta E}{kT}} = 9,123 \cdot 10^{-7}$$

b) $T = 2900 \text{ K}$

$$\mathbf{Na^+} \quad \frac{N_j}{N_o} = \frac{g_j}{g_o} e^{-\frac{\Delta E}{kT}} = 6,555 \cdot 10^{-4} \quad \mathbf{Mg^{2+}} \quad \frac{N_j}{N_o} = \frac{g_j}{g_o} e^{-\frac{\Delta E}{kT}} = 5,727 \cdot 10^{-5}$$

c) $T = 6000 \text{ K}$

$$\mathbf{Na^+} \quad \frac{N_j}{N_o} = \frac{g_j}{g_o} e^{-\frac{\Delta E}{kT}} = 0,051 \quad \mathbf{Mg^{2+}} \quad \frac{N_j}{N_o} = \frac{g_j}{g_o} e^{-\frac{\Delta E}{kT}} = 0,016$$