Nuklearna fizika
- vježbe -

2. Nukleosinteza

Korisne konstante

$$u = a.m.u. = 931.50\,\mathbf{MeV}\,/\,c^2$$

$$m_p = 1.00728\,\mathbf{u} = 938.28\,\mathbf{MeV}\,/\,c^2$$

$$m_n = 1.00867\,\mathbf{u} = 939.57\,\mathbf{MeV}\,/\,c^2$$

$$m_d = 2.01355\,\mathbf{u} = 1875.63\,\mathbf{MeV}\,/\,c^2$$

$$m_\alpha = 4.001506\,\mathbf{u} = 3727.41\,\mathbf{MeV}\,/\,c^2$$

$$m_e = 511.003\,\mathbf{keV}\,/\,c^2$$

$$h = 4.1357 \cdot 10^{-21}\,\mathbf{MeV} \cdot \mathbf{s}$$

$$\hbar c = 197.319\,\mathbf{MeV} \cdot \mathbf{fm}$$

$$k = 8.6174 \cdot 10^{-05}\,\mathbf{eV}\,/\mathbf{K}$$

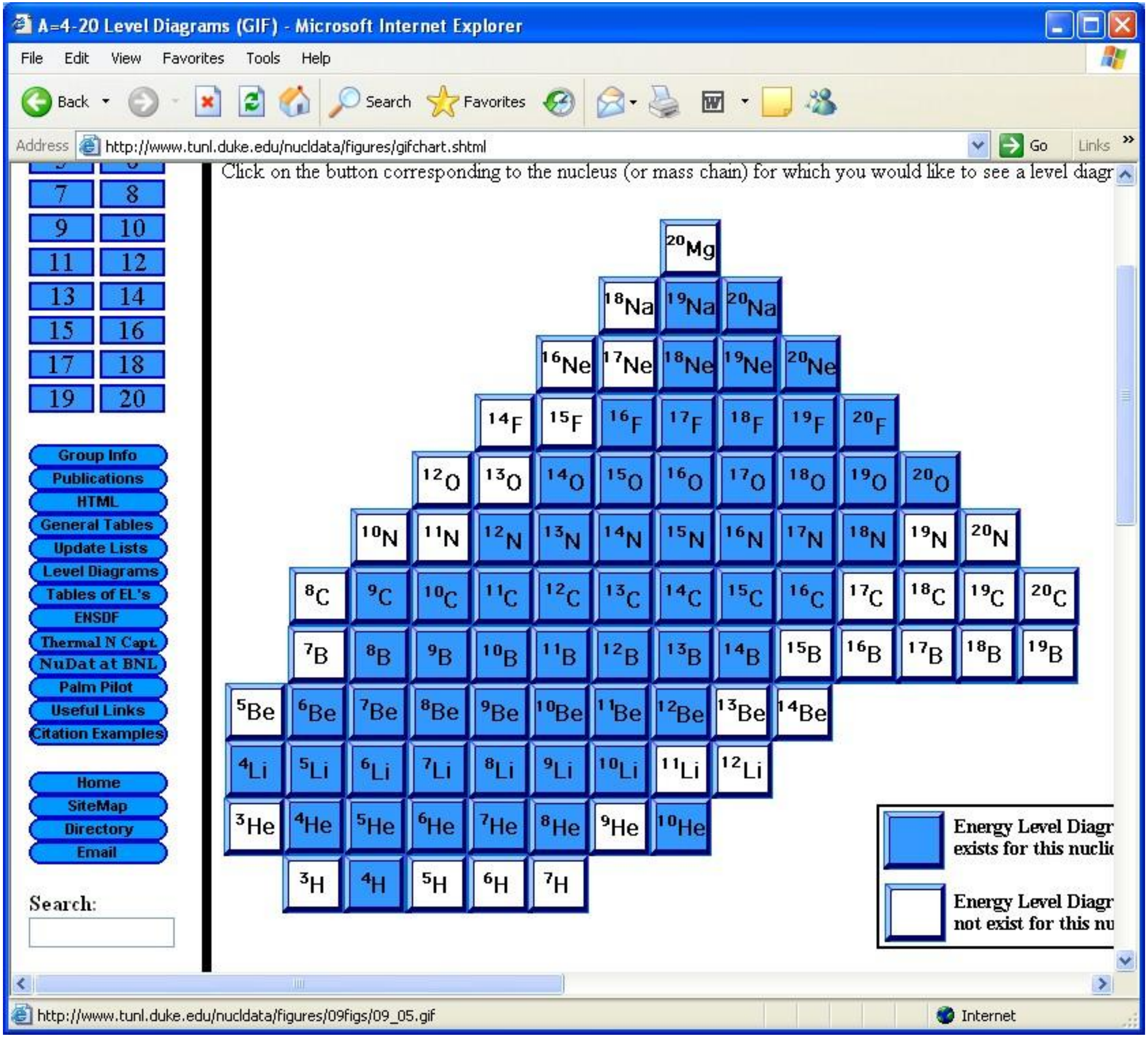
Korisni linkovi

Baze podataka

- A=4-20
- Nuclear Levels and Gammas Search
- National Nuclear Data Center
- Atomic Masses, Q-values, ...
- EXFOR: Exp. Nucl. Reaction Data
- LANL Q-values
- LUND Q-value calculator
- BNL Q-value calculator

Traženje članaka

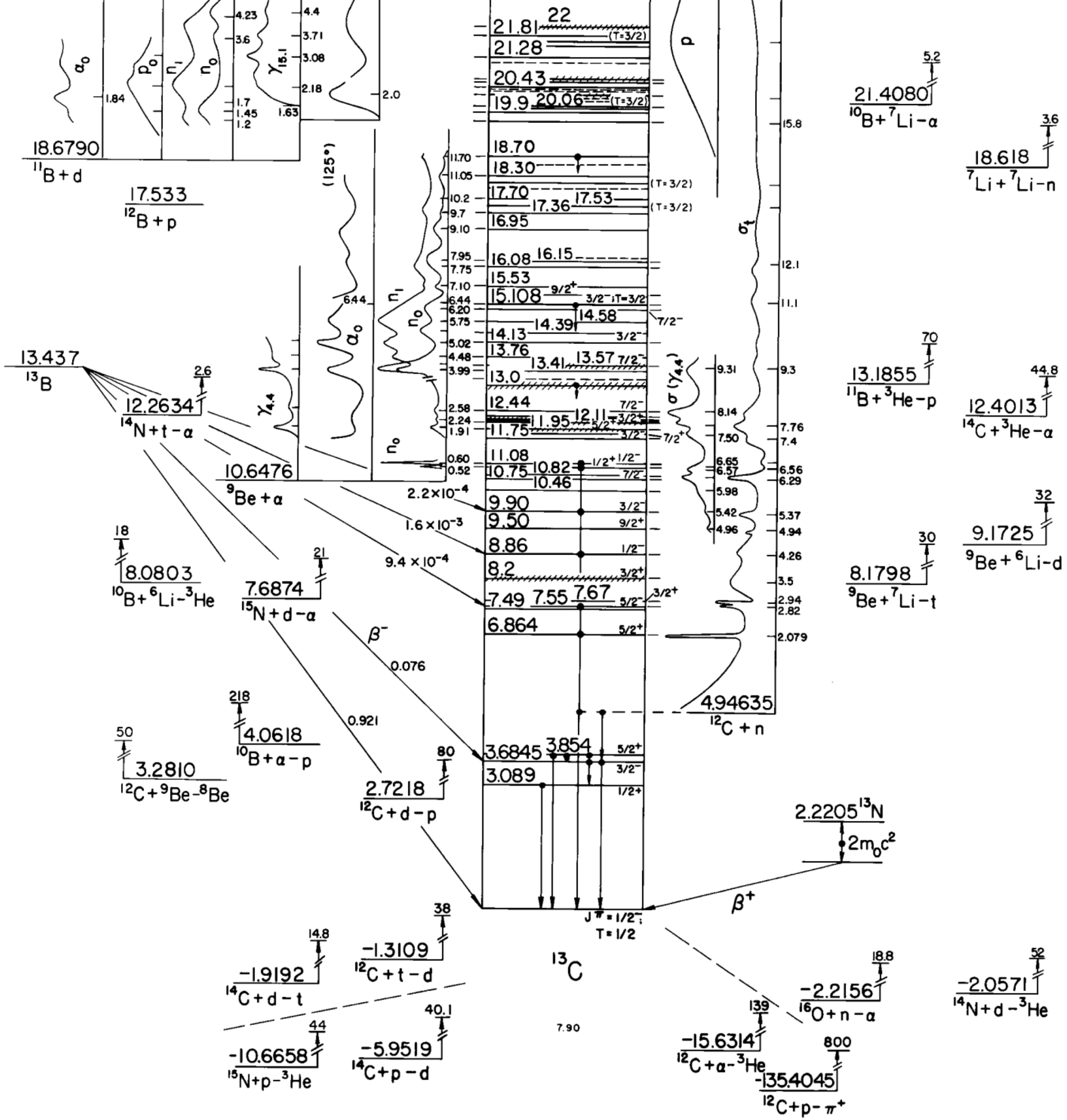
- Nuclear Science References
- The Isotope Explorer NSR Server
- KEK preprints
- SLAC SPIRES
- CERN - CDS Search
- arXiv - Nuclear Theory
- arXiv - Nuclear Experiment
- Web of Science



Click on the button corresponding to the nucleus (or mass chain) for which you would like to see a level diagr

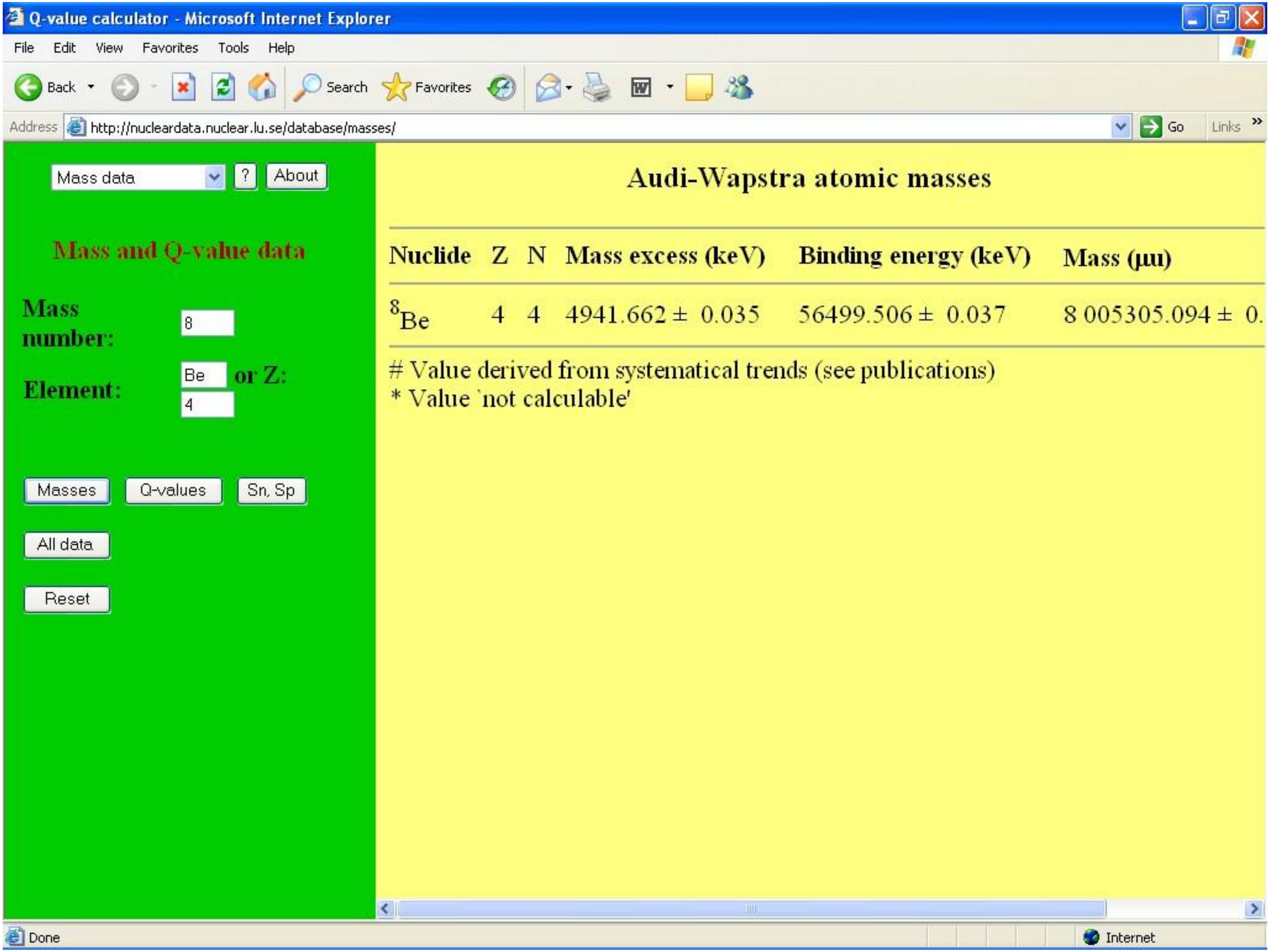
							⬜ ^{20}Mg						
						⬜ ^{18}Na	🟦 ^{19}Na	🟦 ^{20}Na					
					⬜ ^{16}Ne	⬜ ^{17}Ne	🟦 ^{18}Ne	🟦 ^{19}Ne	🟦 ^{20}Ne				
				⬜ ^{14}F	⬜ ^{15}F	🟦 ^{16}F	🟦 ^{17}F	🟦 ^{18}F	🟦 ^{19}F	🟦 ^{20}F			
			⬜ ^{12}O	⬜ ^{13}O	🟦 ^{14}O	🟦 ^{15}O	🟦 ^{16}O	🟦 ^{17}O	🟦 ^{18}O	🟦 ^{19}O	🟦 ^{20}O		
		⬜ ^{10}N	⬜ ^{11}N	🟦 ^{12}N	🟦 ^{13}N	🟦 ^{14}N	🟦 ^{15}N	🟦 ^{16}N	🟦 ^{17}N	🟦 ^{18}N	⬜ ^{19}N	⬜ ^{20}N	
	⬜ ^{8}C	🟦 ^{9}C	🟦 ^{10}C	🟦 ^{11}C	🟦 ^{12}C	🟦 ^{13}C	🟦 ^{14}C	🟦 ^{15}C	🟦 ^{16}C	⬜ ^{17}C	⬜ ^{18}C	⬜ ^{19}C	⬜ ^{20}C
	⬜ ^{7}B	🟦 ^{8}B	🟦 ^{9}B	🟦 ^{10}B	🟦 ^{11}B	🟦 ^{12}B	🟦 ^{13}B	🟦 ^{14}B	⬜ ^{15}B	⬜ ^{16}B	⬜ ^{17}B	⬜ ^{18}B	⬜ ^{19}B
⬜ ^{5}Be	🟦 ^{6}Be	🟦 ^{7}Be	🟦 ^{8}Be	🟦 ^{9}Be	🟦 ^{10}Be	🟦 ^{11}Be	🟦 ^{12}Be	⬜ ^{13}Be	⬜ ^{14}Be				
🟦 ^{4}Li	🟦 ^{5}Li	🟦 ^{6}Li	🟦 ^{7}Li	🟦 ^{8}Li	🟦 ^{9}Li	🟦 ^{10}Li	⬜ ^{11}Li	⬜ ^{12}Li					
⬜ ^{3}He	🟦 ^{4}He	🟦 ^{5}He	🟦 ^{6}He	🟦 ^{7}He	🟦 ^{8}He	⬜ ^{9}He	🟦 ^{10}He						
	⬜ ^{3}H	🟦 ^{4}H	⬜ ^{5}H	⬜ ^{6}H	⬜ ^{7}H								

🟦	Energy Level Diagr exists for this nuclid
⬜	Energy Level Diagr not exist for this nu



¹³C
Jπ = 1/2⁻; T = 1/2
4.94635
¹²C + n
3.6845 5/2⁺
3.854
3.089 3/2⁻
1/2⁺
6.864 5/2⁺
7.49 7.55 7.67 5/2⁻ 3/2⁺
8.2 3/2⁺
8.86 1/2⁻
9.50 9/2⁺
9.90 3/2⁻
10.46
10.75 10.82 7/2⁻
11.08 1/2⁺ 1/2⁻
11.75 3/2⁻ 7/2⁺
11.95 5/2⁺
12.11 3/2⁺
12.44 7/2⁻
13.0
13.41 13.57 7/2⁻
13.76
14.13 14.39 3/2⁻
14.58 7/2⁻
15.108 9/2⁺ 3/2⁻; T=3/2
15.53
16.08 16.15
16.95
17.36 17.53 (T=3/2)
17.70 (T=3/2)
18.30
18.70
19.9 20.06 (T=3/2)
20.43
21.28
21.81 22 (T=3/2)
18.6790
¹¹B + d
17.533
¹²B + p
13.437
¹³B
12.2634 2.6
¹⁴N + t − α
10.6476
⁹Be + α
2.2×10⁻⁴
1.6×10⁻³
9.4×10⁻⁴
8.0803 18
¹⁰B + ⁶Li − ³He
7.6874 21
¹⁵N + d − α
β⁻
0.076
0.921
4.0618 218
¹⁰B + α − p
3.2810 50
¹²C + ⁹Be − ⁸Be
2.7218 80
¹²C + d − p
−1.3109 38
¹²C + t − d
−1.9192 14.8
¹⁴C + d − t
−10.6658 44
¹⁵N + p − ³He
−5.9519 40.1
¹⁴C + p − d
21.4080 5.2
¹⁰B + ⁷Li − α
18.618 3.6
⁷Li + ⁷Li − n
13.1855 70
¹¹B + ³He − p
12.4013 44.8
¹⁴C + ³He − α
9.1725 32
⁹Be + ⁶Li − d
8.1798 30
⁹Be + ⁷Li − t
2.2205 ¹³N
2m₀c²
β⁺
−2.2156 18.8
¹⁶O + n − α
−2.0571 52
¹⁴N + d − ³He
−15.6314 139
¹²C + α − ³He
−135.4045 800
¹²C + p − π⁺
α₀ p₀ n₁ n₀ γ₁₅.₁
1.84 4.23 3.6 1.7 1.45 1.2 1.63
4.4 3.71 3.08 2.18 2.0
(125°)
γ₄.₄ α₀ n₁ n₀ 6.44
11.70 11.05 10.2 9.7 9.10 7.95 7.75 7.10 6.44 6.20 5.75 5.02 4.48 3.99 2.58 2.24 1.91 0.60 0.52
p
σₜ
15.8 12.1 11.1 9.3 7.76 7.4 6.56 6.29 5.37 4.94 4.26 3.5 2.94 2.82 2.079
σ(γ₄.₄)
9.31 8.14 7.50 6.65 6.57 5.98 5.42 4.96

7.90



Mass number: 8

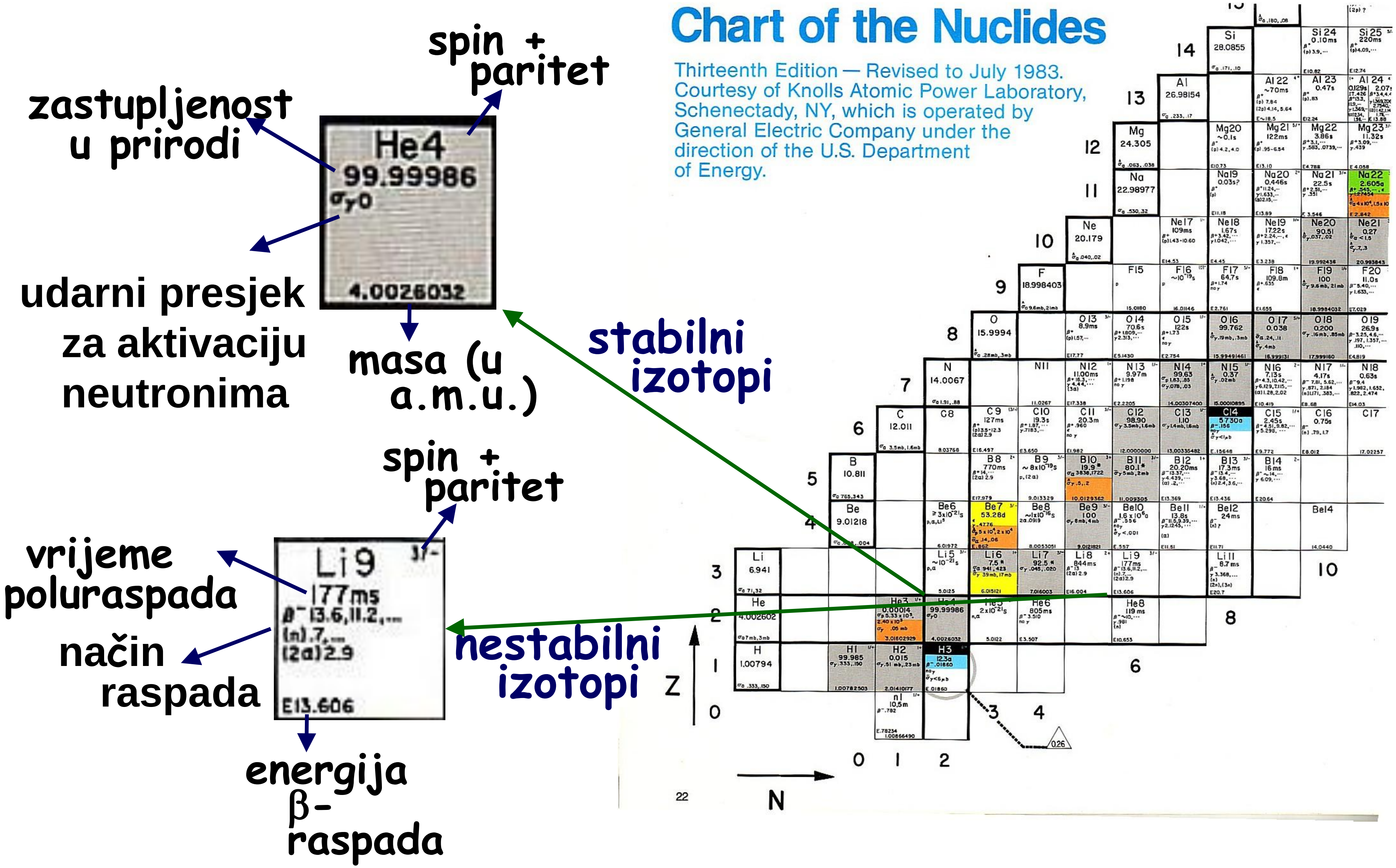
Element: Be or Z: 4

Audi-Wapstra atomic masses

Nuclide	Z	N	Mass excess (keV)	Binding energy (keV)	Mass (μu)
^{8}Be	4	4	4941.662 ± 0.035	56499.506 ± 0.037	8 005305.094 ± 0.

Value derived from systematical trends (see publications)
* Value `not calculable'

Tablice izotopa



spin + paritet

zastupljenost u prirodi

udarni presjek za aktivaciju neutronima

masa (u a.m.u.)

stabilni izotopi

spin + paritet

vrijeme poluraspada

način raspada

energija β- raspada

nestabilni izotopi

Chart of the Nuclides

Thirteenth Edition — Revised to July 1983.
Courtesy of Knolls Atomic Power Laboratory,
Schenectady, NY, which is operated by
General Electric Company under the
direction of the U.S. Department
of Energy.

Zadatak 9. Izračunajte odbojnu kulonsku potencijalnu energiju za:
a) dva protona na udaljenosti 1 fm,
b) jezgru zlata (Z=79) i α-česticu na udaljenosti 10 fm i
c) dvije jezgre srebra (Z=47, A=107) koje su vrlo blizu, ali se ne preklapaju

---

Rješenje 9.

$$V = \frac{Z_1 Z_2 e^2}{d}$$

$$V\ (\mathbf{MeV}) = \alpha \hbar c \frac{Z_1 Z_2}{d} = \frac{197.3}{137.04}\ (\mathbf{MeV\ \ fm})\ \frac{Z_1 Z_2}{d\ (\mathbf{fm})} = 1.44\ (\mathbf{MeV\ \ fm})\ \frac{Z_1 Z_2}{d\ (\mathbf{fm})}$$

a) $Z_1=Z_2=1$, d=1fm ⇨ V= 1.44 MeV

b) $Z_1=79$, $Z_2=2$, d=10fm ⇨ V= 22.7 MeV

c) $R=1.2\times A^{1/3}$ fm, d=2R= 11.4 fm

$Z_1=Z_2=47$ ⇨ V= 279.0 MeV

Zadatak 10. Izrazite "*kt*" preko "nuklearnih" jedinica (T_6, MeV)

Rješenje 10.

$$kT = 1.38\cdot 10^{-23} T \ \mathbf{J} =$$

$$= \frac{1.38\cdot 10^{-23}}{1.6\cdot 10^{-19}\cdot 10^{3}} 10^{6}\,\mathbf{keV} =$$

$$= 8.62\cdot 10^{-2}\ T_6\ \mathbf{keV}$$

- Zašto je to bitno? Jer je raspodjela po brzina u nerelativističkoj, nedegeneriranoj plazmi dana Maxwell-Boltzmannovom formulom:

$$\phi(v) = 4\pi\left(\frac{\mu}{2\pi kT}\right)^{3/2} v^2 \exp\left(-\frac{\mu v^2}{2kT}\right)$$

gdje je

$$\mu = \frac{m_p m_T}{m_p + m_T}$$

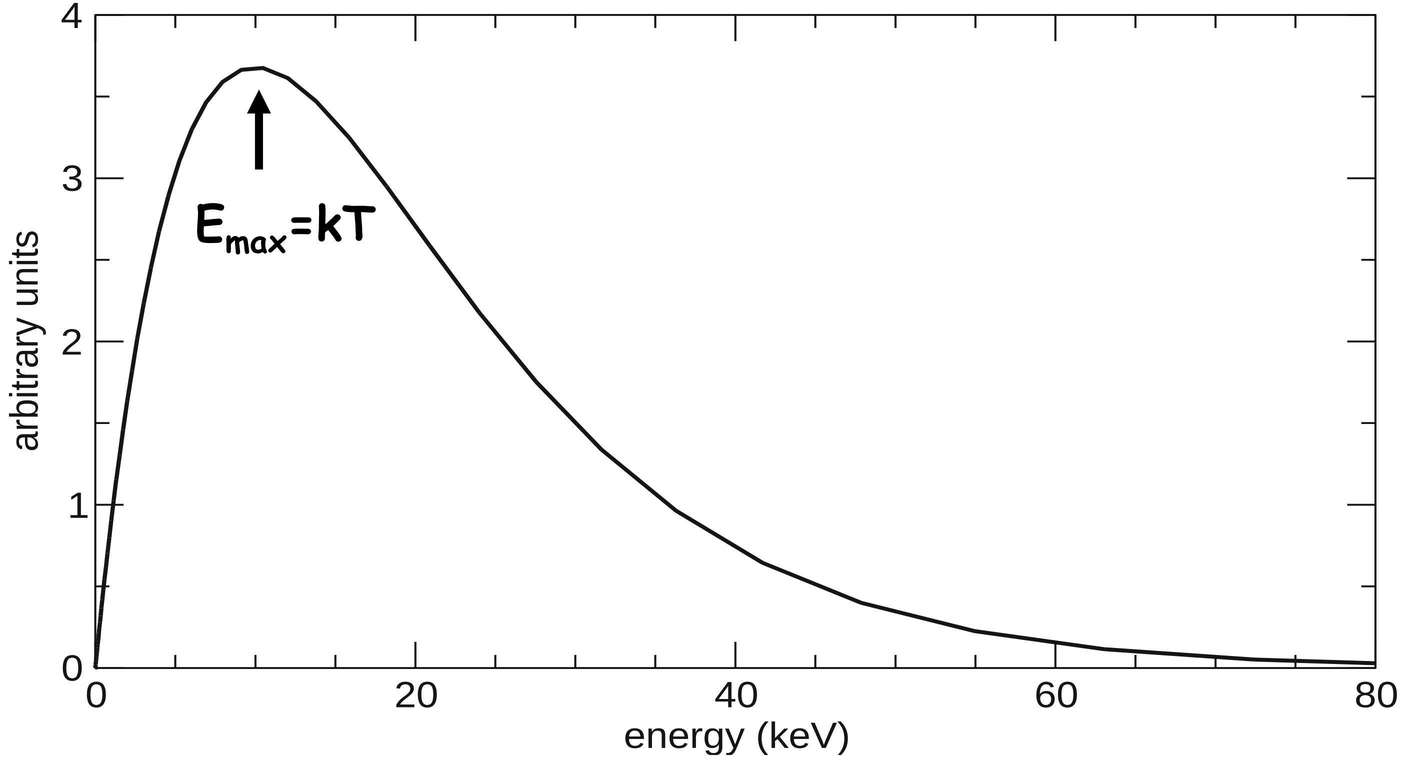
- za središte Sunca:

$$T = 15\cdot 10^{6}\ \mathbf{K} \quad \Rightarrow \quad kT \approx 1\ \mathbf{keV}$$

Termonuklearne reakcije

- u zvijezdama se ioni nalaze u termičkoj reavnoteži, pa je njihova raspodjela po brzinama dobro opisana Mawell-Boltzmannovim izrazom:

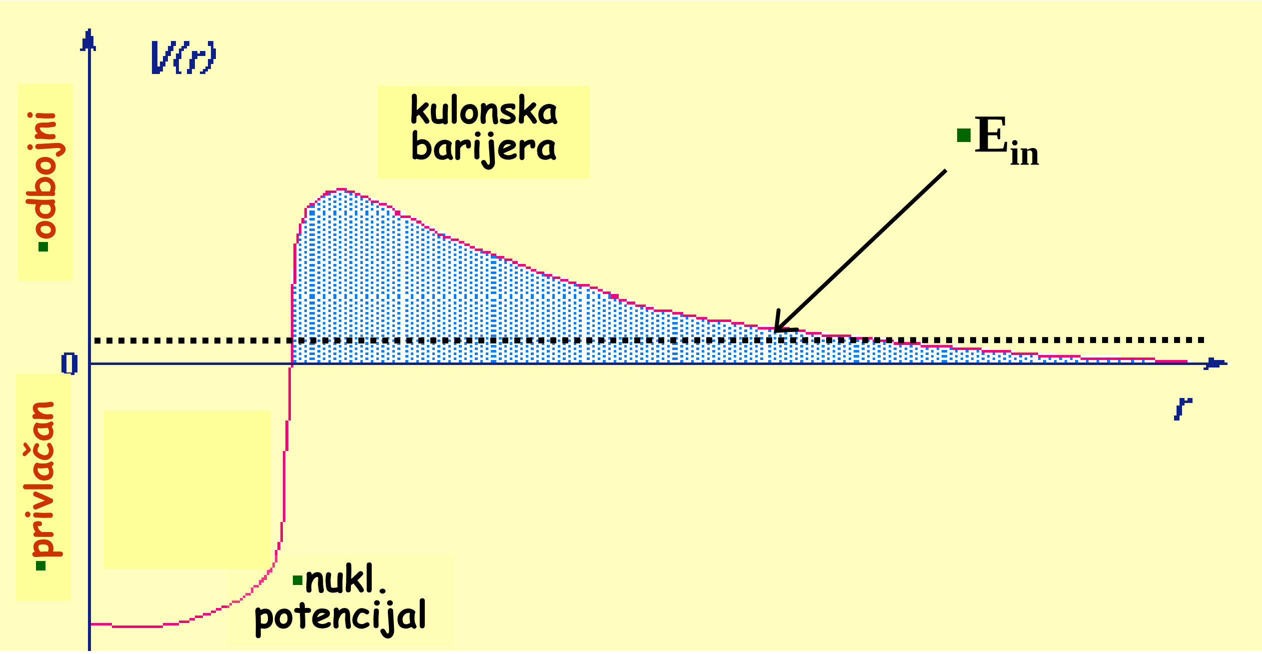
$$\Phi(v) = 4\pi \left(\frac{\mu}{2\pi kT} \right)^{3/2} v^2 \exp\left(-\frac{\mu v^2}{2kT} \right) \quad \text{gdje je} \quad \mu = \frac{m_a m_A}{m_a + m_A}$$



$E_{max}=kT$

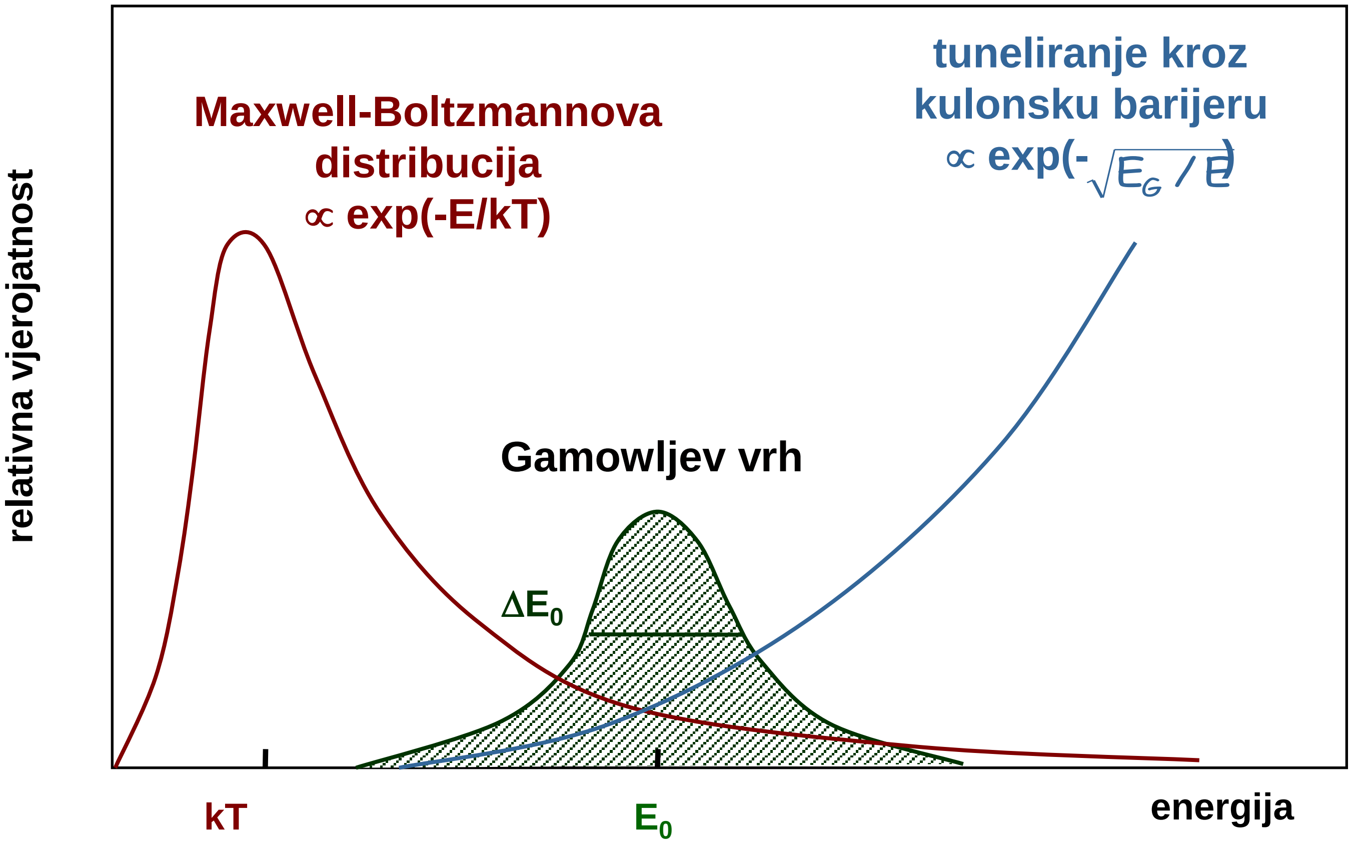
Nuklearne reakcije: energije

- u zvijezdama nema slobodnih neutrona jer su β-nestabilni ($T_{1/2}$= 10 min)
- fuzija se u zvijezdama odvija između nabijenih jezgara!
- nuklearna sila je jaka, ali vrlo kratkog dosega
- da bi se nabijene jezgre uopće dovoljno približile da bi nuklearna sila među njima počela djelovati, moraju savladati kulonsku barijeru



Zadatak 11. Izvedite izraz za Gamowljevu energiju, tj. energiju na kojoj je vjerojatnost odigravanja termonuklearnih reakcija maksimalna.

Rješenje 11.



Rješenje 11.

Brzina odvijanja reakcije dana je s:

$$r = N_x N_Y \langle \sigma(v) v \rangle$$

Vjerojatnost reakcije možemo faktorizirati kao:

$$P(\text{reakcije}) = P(\text{tuneliranja}) \cdot P(\text{apsorpcije})$$

$$P(\text{tuneliranja}) = e^{-\frac{4\pi}{h}\int\left[2m\left(\frac{Z_1 Z_2 e^2}{r}\right)\right]^{1/2} dr}$$

$$P(\text{tuneliranja}) = e^{-2\pi\eta}$$

gdje je η Sommerfeldov parametar defeniran s:

$$\eta = \frac{2\pi Z_1 Z_2 e^2}{hv}$$

Rješenje 11.

Udarni presjek se parametrizira uvođenjem "S-faktora":

$$\sigma(E) = \frac{S(E)}{E} e^{-2\pi\eta}$$

Raspodjela jezgara po brzinama (energijama) opisana je Maxwell-Boltzmanovom distribucijom:

$$\langle \sigma v \rangle = \int_0^\infty v\sigma(v)\left(\frac{\mu}{2\pi kT}\right)^{3/2} e^{-\mu v^2/2kT} d^3 v =$$

$$= 4\pi\left(\frac{\mu}{2\pi kT}\right)^{3/2} \int_0^\infty v^3 \sigma(v) e^{-\mu v^2/2kT} dv =$$

$$= \left(\frac{8}{\pi\mu}\right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty E\sigma(v) e^{-E/kT} dE$$

Rješenje 11.

Iskoristimo li malo prije definiran S-faktora, dobivamo:

$$\langle \sigma v \rangle = \sqrt{8/\pi\mu}\,(1/kT)^{3/2} \int S(E) e^{-E/kT - b/\sqrt{E}} dE$$

gdje je *b* dan s:

$$b = 2\pi\eta\sqrt{E} =$$

$$= 31.28 \cdot Z_1 Z_2 A^{1/2} \sqrt{\mathrm{keV}}$$

Integrand u izrazu za <σv> bit će maksimalan kada je:

$$\frac{E}{kT} + \frac{b}{\sqrt{E}} = \min$$

Rješenje 11.

Tražimo minimum deriviranjem:

$$\frac{1}{kT}-\frac{1}{2}\frac{b}{E^{3/2}}=0$$

i on se postiže za energiju:

$$E_G=\left(\frac{bkT}{2}\right)^{2/3}=$$

$$=1.220\left(Z_1^2 Z_2^2 A T_6^2\right)^{1/3}\ \text{keV}$$

Dobivena energija (ili tzv. Gamowljeva energija) je "optimalna" upadna energija za odvijanje termonuklearnih reakcija...

Zadatak 12. Izračunajte Gamowljevu energiju za reakcije p+p i α+α na temperaturi središta Sunca

Rješenje 12.

$$E_G = 1.220 \cdot \left(Z_1^2 Z_2^2 A T_6^2\right)^{1/3} \text{ keV}$$

$$\Delta E = \frac{4}{\sqrt{3}} \sqrt{E_0 k T} = 0.237 \left(Z_1^2 Z_2^2 A \right)^{1/6} T_9^{5/6} \quad \text{MeV}$$

$$A = \frac{A_p A_T}{A_p + A_T}$$

- za $Z_1=Z_2=1$, $A=1/2$,

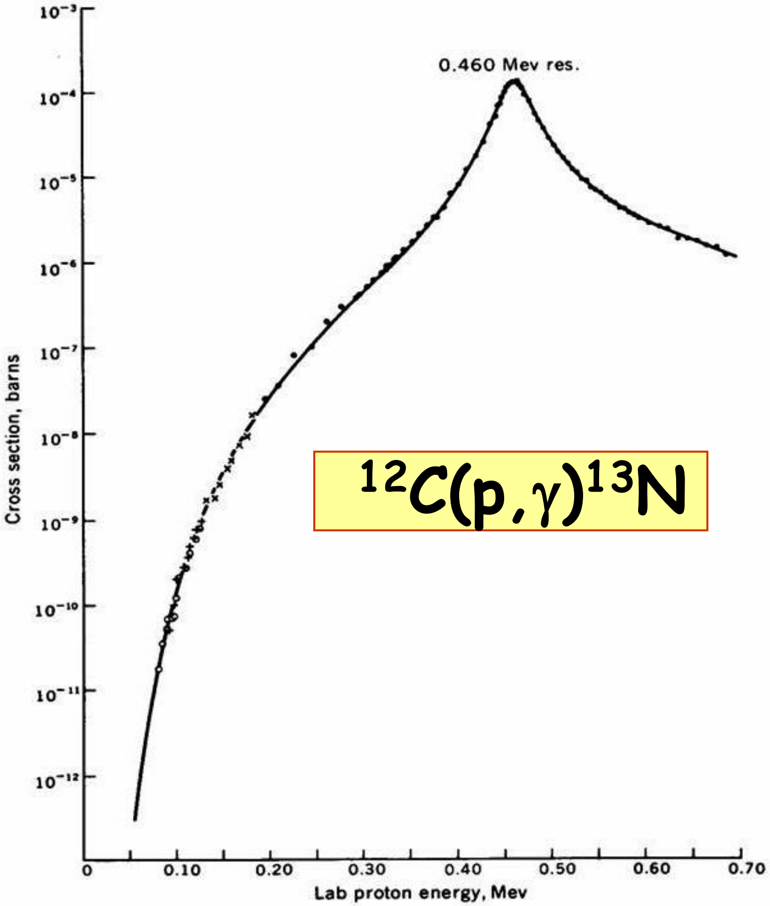
  $T=15\times10^6$ K ⇨ $E_G=$ 5.9 keV

- za $Z_1=Z_2=2$, $A=2$,

  $T=15\times10^6$ K ⇨ $E_G=$ 23.5 keV

Ovisnost nukl. reakcija o energiji

- upadna energija (brzina) čestica je parametar koji ponajviše određuje brzinu odvijanja nuklearnih reakcija!
- dakako, bitni su i struktura jezgara, mehanizam reakcije itd.
- primjer:



$^{12}C(p,\gamma)^{13}N$

Ovisnost nukl. reakcija o energiji

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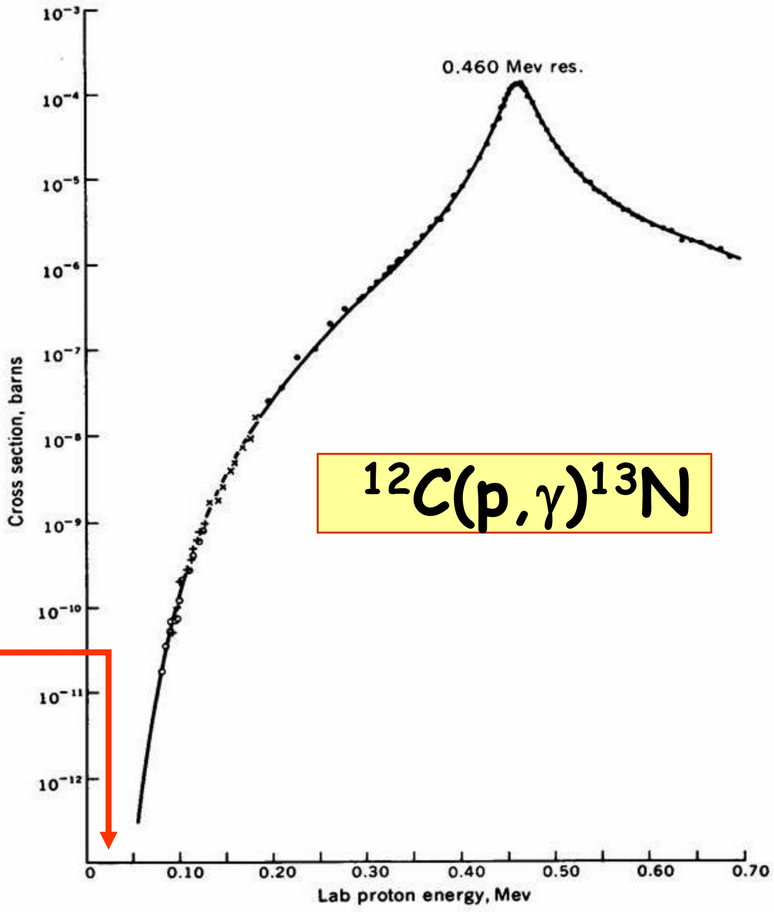


$^{12}C(p,\gamma)^{13}N$

udarni presjek trebamo ovdje!

S-faktor nuklearne reakcije

- budući da u laboratoriju zbog niskog udarnog presjeka najčešće nije moguće direktno mjeriti udarni presjek na traženoj energiji, potrebno je ili napraviti dobar teorijski model ili ekstrapolirati eksperimentalne rezultate na višim energijama

- ekstrapolacija je lakša ako se radi na veličini iz koje je izdvojen efekt tuneliranja (jer on vodi na brz pad udarnog presjeka na niskim energijama)

  ⇨ definira se nova veličina, tzv. S-faktor:

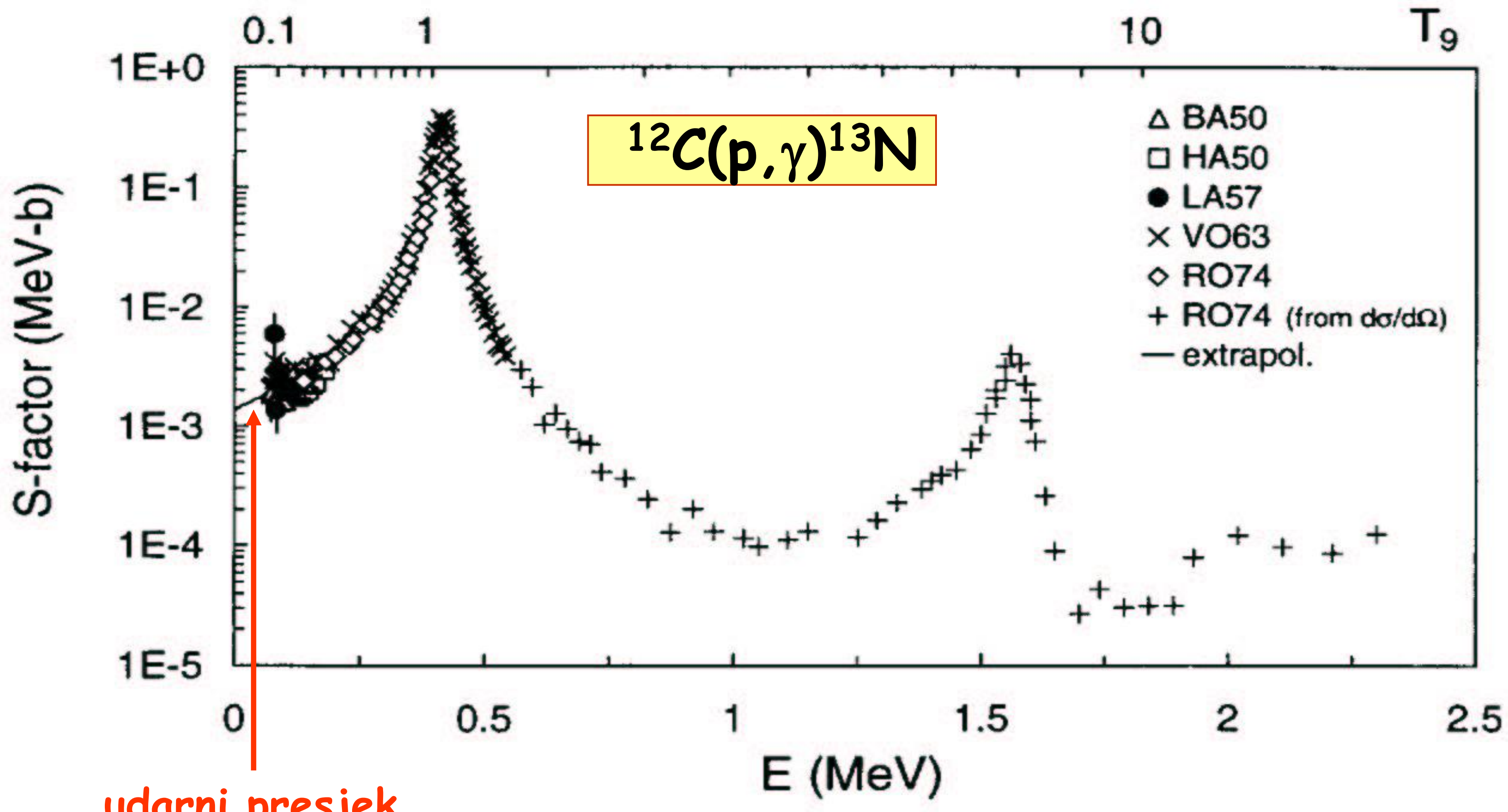
$$S(E) = \sigma(E) e^{2\pi Z_1 Z_2 e^2 / \hbar v}$$

- S-faktor i dalje uključuje svu bitnu nuklearnu fiziku: efekte strukture jezgara koje sudjeluju u reakciji, mehanizma reakcije i dr.

- ekstrapolacija S-faktora na niske energije dosta uspješna, osim u slučaju postojanja rezonanci!

S-faktor za $^{12}C(p,\gamma)^{13}N$

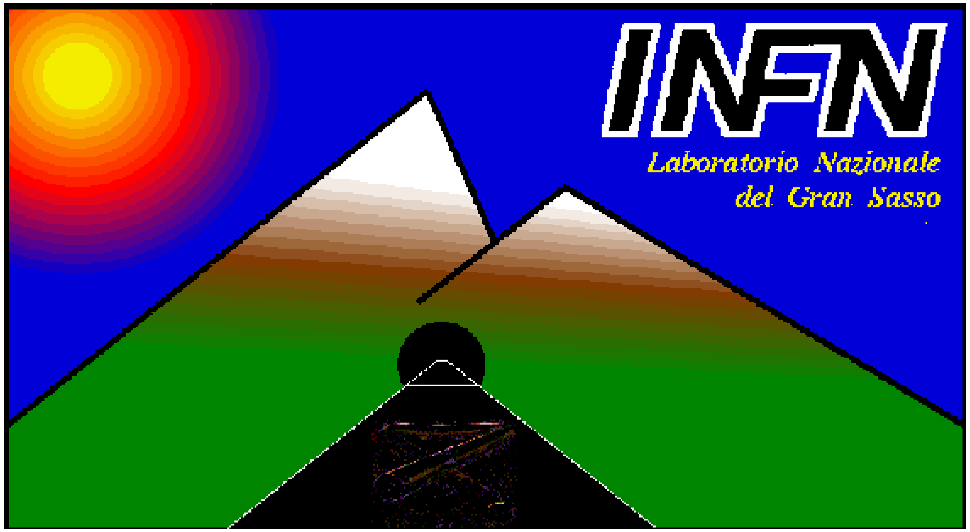
- za prethodni primjer imamo:



udarni presjek
trebamo ovdje!

LUNA

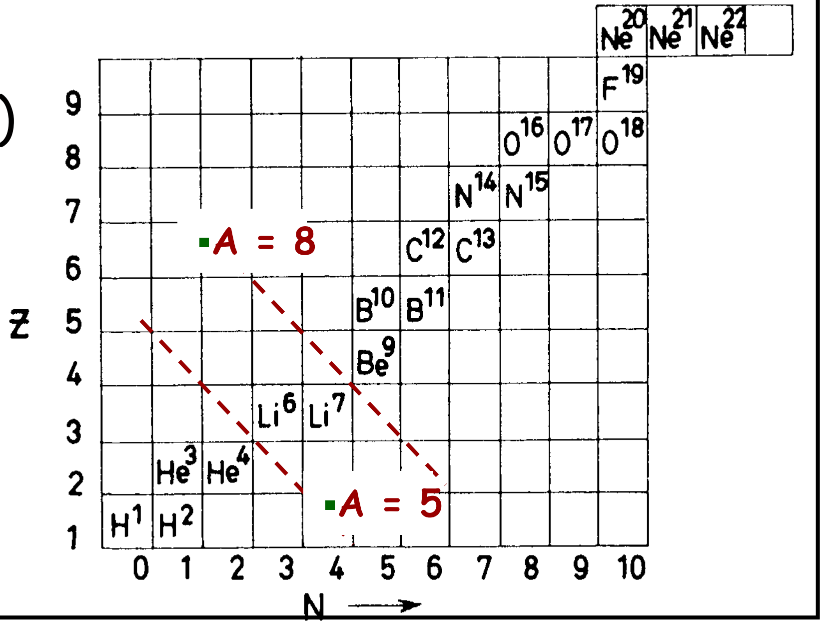
- od *engl.* Laboratory Underground for Nuclear Astrophysics
- tunel Gran Sasso, Italija
- 1400 m ispod površine zemlje
- tok kozmičkog zračenja milion puta manji nego na površini



- 1998. - prvo mjerenje reakcije $^{3}\mathrm{He}+^{3}\mathrm{He} \rightarrow ^{4}\mathrm{He}+2p$ na Gamowljevoj energiji
- detektiran u prosjeku 1 događaj mjesečno!
- od tada izmjerene još tri reakcije i u dva slučaja dobiveni bitno drugačiji rezultati u odnosu na prijašnje ekstrapolacije i teorijska predviđanja...

Gorenje vodika

- nakon prve tri minute od Velikog Praska, vidljiva materija u Svemiru sastojala se od ≈75% vodika i ≈25% helija (maseni udjeli)
- u preostalih 13.7 milijardi godina, materija je prerađivan u zvijezdama, ali taj omjer nije bitno promijenjen (u Sunčevom sustavu: <2% nuklida ima *A*>5)
- dakle, nuklearne reakcije kreću u smjesi helija i vodika; nema neutrona!
- vjerojatnost nuklearne reakcije između 3 ulazne čestice zanemarivo malena ⇨ ne možemo direktno 4 protona pretvoriti u alfa-česticu (jezgru ^{4}He)
- dodatni problem: ne postoje čestično stabilne jezgre s *A*=5 i *A*=8



Gorenje vodika

Korak 1: $\xcancel{p+p\rightarrow {}^{2}He}$

^{2}He nije stabilan

$$\boxed{p+p\rightarrow d+e^{+}+\nu_{e}}$$

Korak 2: $\boxed{d+p\rightarrow {}^{3}He+\gamma}$

$$\xcancel{d+d\rightarrow {}^{4}He}$$

deuterona premalo!

Korak 3: $\xcancel{{}^{3}He+p\rightarrow {}^{4}Li}$

^{4}Li nije stabilan

$$\xcancel{{}^{3}He+d\rightarrow {}^{4}He+n}$$

deuterona premalo!

$$\boxed{{}^{3}He+{}^{3}He\rightarrow {}^{4}He+2p}$$

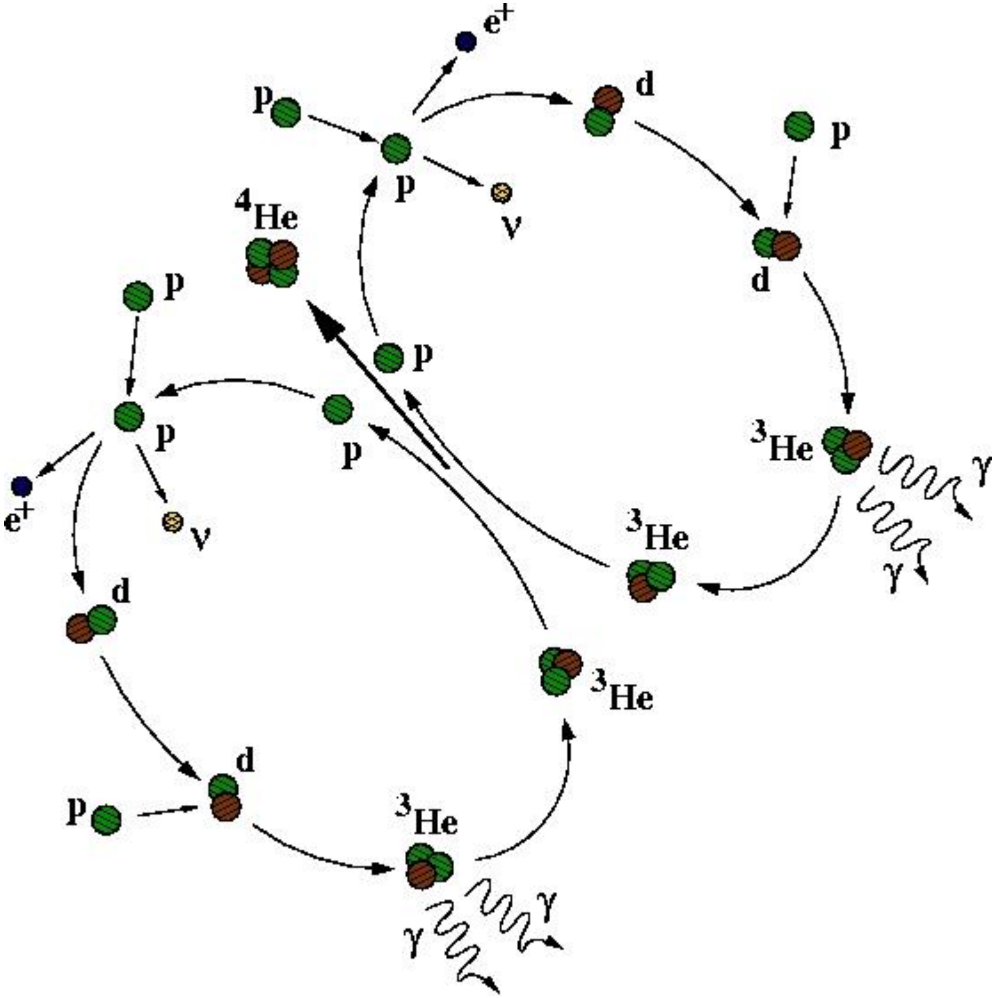
- za razliku od deuterona koji se po nastanku brzo razbija fotonima, ^{3}He je prilično stabilan i zato je zadnji korak moguć!

Zadatak 13. Za *pp*-ciklus odredite ukupnu oslobođenu energiju (srednja neutrinska energija iz raspada pozitrona je 0.26 MeV)

Rješenje 13.

- *pp*-ciklus ilustriran je na slici; reakcije su:

	Q / MeV
$p+p\rightarrow d+e^{+}+\nu$	**1.442**
$d+p\rightarrow {}^{3}\mathrm{He}+\gamma$	**5.493**
${}^{3}\mathrm{He}+{}^{3}\mathrm{He}\rightarrow {}^{4}\mathrm{He}+2p$	**12.859**



- ukupan Q:
  2×1.442+2×5.493
  +12.859-2×0.26≈
  ≈ 26.2 MeV

- oslobađa se ≈ 26.2 MeV po svakoj stvorenoj jezgri ^{4}He

Zadatak 13. Za *pp*-ciklus odredite ukupnu oslobođenu energiju (srednja neutrinska energija iz raspada pozitrona je 0.26 MeV)

Rješenje 13.

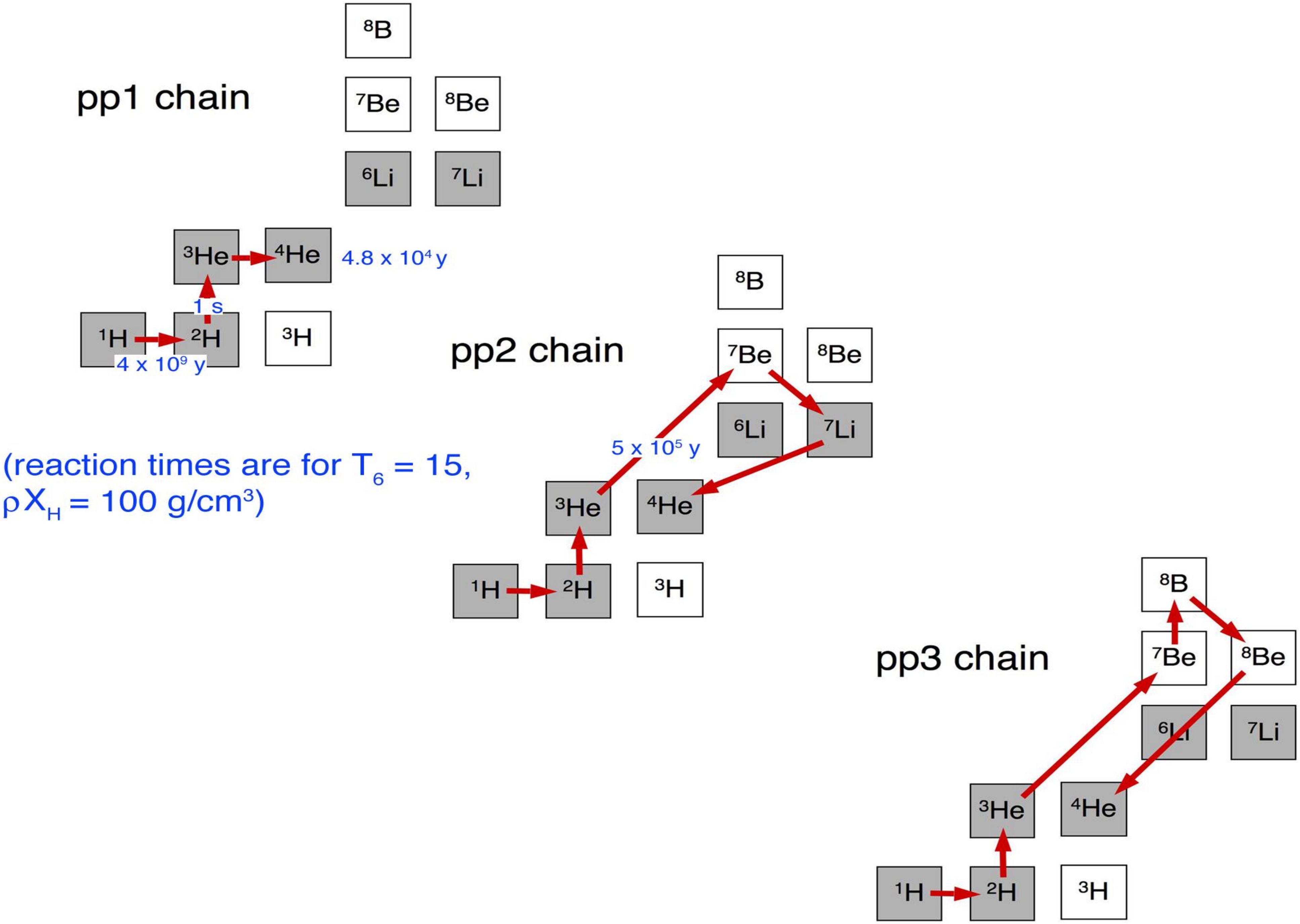
- netto-efekt *pp*-ciklusa je taj da su 4 protona pretvorena u helij, te da su stvorena dva pozitrona i neutrina; rezultat se može dobiti i na drugi način:

$$Q = (4m_p - m_\alpha)\cdot c^2 + 2m_e - 2\cdot 0.26\,\mathbf{MeV} =$$

$$= (4\cdot 1.00728\ - 4.00151\,)\cdot 931.5\,\mathbf{MeV} + 2\cdot 511\,\mathbf{keV} - 0.52\,\mathbf{MeV}\ =$$

$$\approx 26.2\,\mathbf{MeV}$$

- važni su i tzv. slabi prijelazi, pa je *pp*-ciklus vrlo spor!
- na višim temperaturama: *pp*-II, *pp*-III, CNO-ciklus itd.
- eksperimentalan “manjak” broja neutrina ⇨ problem Sunčevih neutrina ⇨ neutrinske oscilacije!



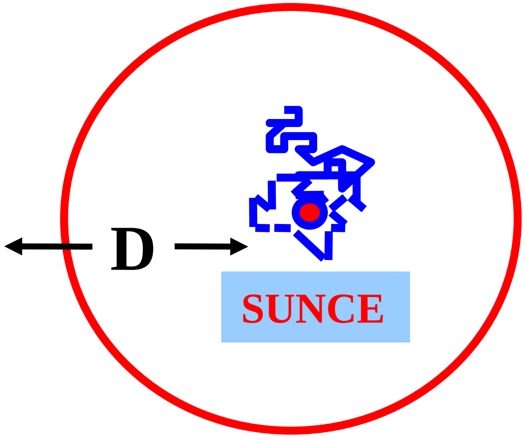
pp1 chain

pp2 chain

pp3 chain

(reaction times are for $T_6 = 15$, $\rho X_H = 100\ \mathrm{g/cm^3}$)

Problem Sunčevih neutrina



- fotoni na svom putu iz Sunca se raspršuju i treba im $\approx 10^6$ godina da izađu!

- neutrini zbog vrlo slabe interakcije nesmetano izlaze i nose informaciju o nuklearnim reakcijama

- R.Davis, 1968.god. – “brojao” Sunčeve neutrine koristeći ogroman volumen napunjeno perkloretilenom

- neutrine detektirao putem inverznog beta-raspada:

$$\nu_e + {}^{37}\mathrm{Cl} \rightarrow e^- + {}^{37}\mathrm{Ar}$$

- umjesto očekivanih 2, eksperimentalno detektirana samo 0.5 događaja po danu

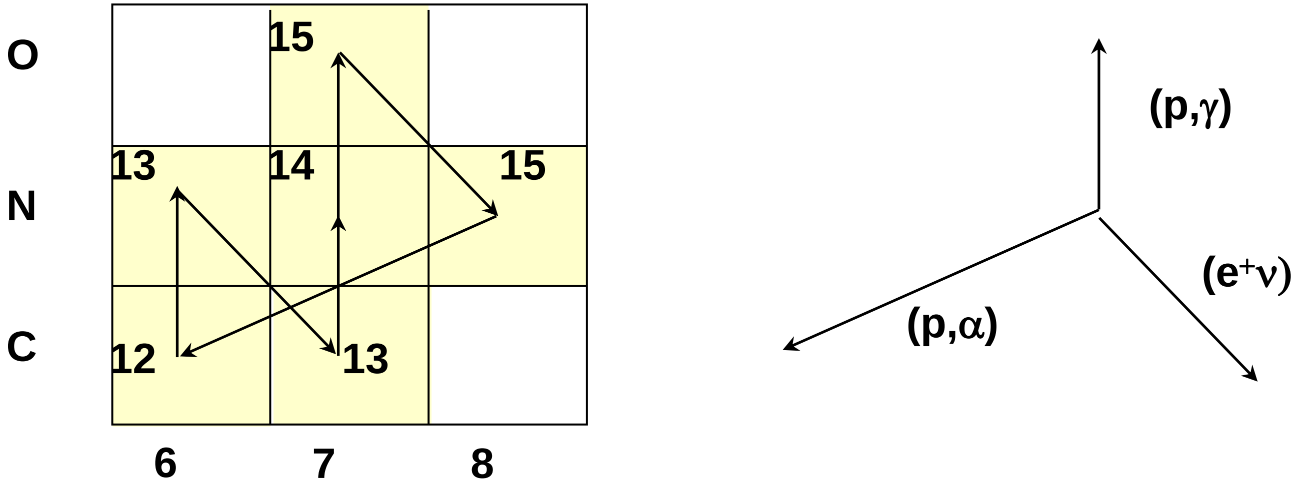
  ⇨ problem Sunčevih neutrina

  (Davis, Nobelova nagrada 2002.)

problem riješen tek nedavno potvrdom postojanja neutrinskih oscilacija (Super-Kamiokande i SNO)

CNO-ciklus (Carbon-Nitrogen-Oxygen)

- na višim temperaturama (T6≈100), najvažniji način sinteze izotopa i energije postaje tzv. CNO-ciklus:

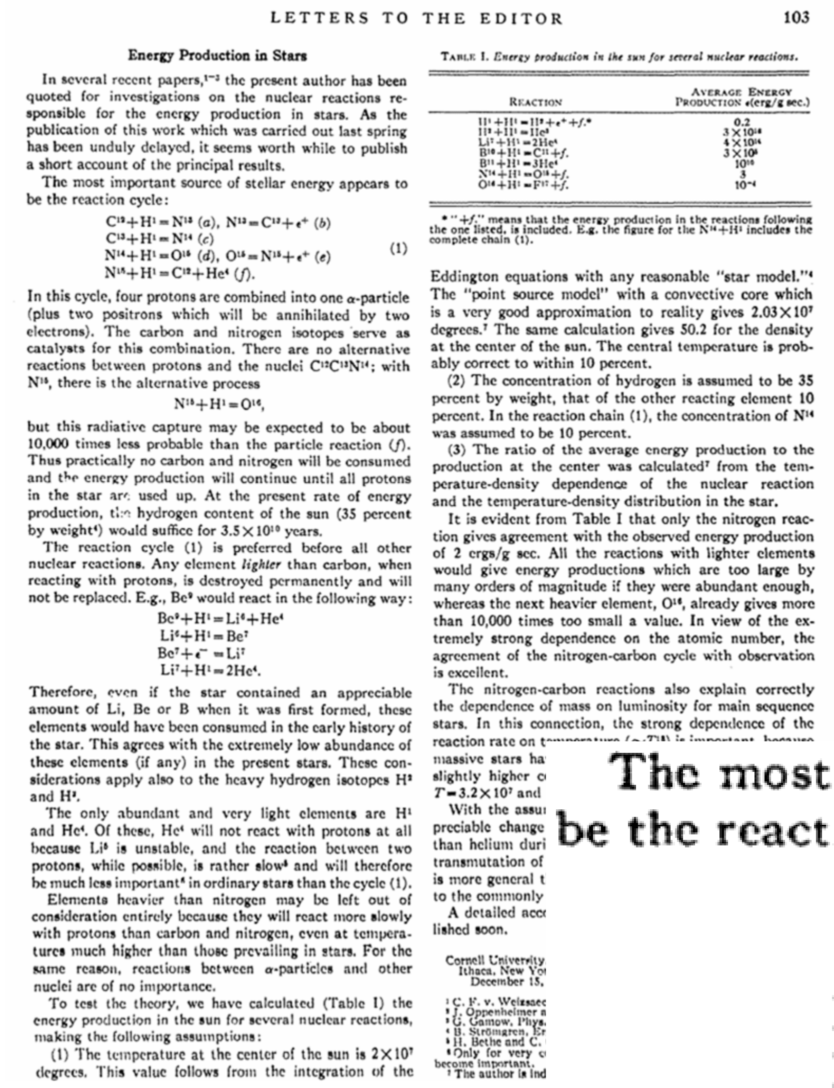


$$^{12}C(p,\gamma)^{13}N(e^{+}\nu)^{13}C(p,\gamma)^{14}N(p,\gamma)^{15}O(e^{+}\nu)^{15}N(p,\alpha)^{12}C$$

- netto-efekt isti kao i za *pp*-ciklus: 4 protona su pretvorena u helij, te da su stvorena dva pozitrona i neutrina
- na još višim temperaturama: "vrući" CNO-ciklus itd.

Nobelova nagrada 1967.

A Scenario-H.A. Bethe (CNO Cycles), Physical Review 55, 103(L) 1939.



Energy Production in Stars

In several recent papers,[1-3] the present author has been quoted for investigations on the nuclear reactions responsible for the energy production in stars. As the publication of this work which was carried out last spring has been unduly delayed, it seems worth while to publish a short account of the principal results.

The most important source of stellar energy appears to be the reaction cycle:

$$\begin{aligned} &C^{12}+H^1=N^{13}\ (a),\quad N^{13}=C^{13}+\epsilon^+\ (b)\\ &C^{13}+H^1=N^{14}\ (c)\\ &N^{14}+H^1=O^{15}\ (d),\quad O^{15}=N^{15}+\epsilon^+\ (e)\\ &N^{15}+H^1=C^{12}+He^4\ (f). \end{aligned} \tag{1}$$

In this cycle, four protons are combined into one α-particle (plus two positrons which will be annihilated by two electrons). The carbon and nitrogen isotopes serve as catalysts for this combination. There are no alternative reactions between protons and the nuclei $C^{12}C^{13}N^{14}$; with N^{15}, there is the alternative process

$$N^{15}+H^1=O^{16},$$

but this radiative capture may be expected to be about 10,000 times less probable than the particle reaction (f). Thus practically no carbon and nitrogen will be consumed and the energy production will continue until all protons in the star are used up. At the present rate of energy production, the hydrogen content of the sun (35 percent by weight[4]) would suffice for 3.5×10^{10} years.

The reaction cycle (1) is preferred before all other nuclear reactions. Any element *lighter* than carbon, when reacting with protons, is destroyed permanently and will not be replaced. E.g., Be^9 would react in the following way:

$$\begin{aligned} &Be^9+H^1=Li^6+He^4\\ &Li^6+H^1=Be^7\\ &Be^7+\epsilon^-=Li^7\\ &Li^7+H^1=2He^4. \end{aligned}$$

Therefore, even if the star contained an appreciable amount of Li, Be or B when it was first formed, these elements would have been consumed in the early history of the star. This agrees with the extremely low abundance of these elements (if any) in the present stars. These considerations apply also to the heavy hydrogen isotopes H^2 and H^3.

The only abundant and very light elements are H^1 and He^4. Of these, He^4 will not react with protons at all because Li^5 is unstable, and the reaction between two protons, while possible, is rather slow[5] and will therefore be much less important[6] in ordinary stars than the cycle (1).

Elements heavier than nitrogen may be left out of consideration entirely because they will react more slowly with protons than carbon and nitrogen, even at temperatures much higher than those prevailing in stars. For the same reason, reactions between α-particles and other nuclei are of no importance.

To test the theory, we have calculated (Table I) the energy production in the sun for several nuclear reactions, making the following assumptions:

(1) The temperature at the center of the sun is 2×10^7 degrees. This value follows from the integration of the Eddington equations with any reasonable "star model."[4] The "point source model" with a convective core which is a very good approximation to reality gives 2.03×10^7 degrees.[7] The same calculation gives 50.2 for the density at the center of the sun. The central temperature is probably correct to within 10 percent.

(2) The concentration of hydrogen is assumed to be 35 percent by weight, that of the other reacting element 10 percent. In the reaction chain (1), the concentration of N^{14} was assumed to be 10 percent.

(3) The ratio of the average energy production to the production at the center was calculated[7] from the temperature-density dependence of the nuclear reaction and the temperature-density distribution in the star.

It is evident from Table I that only the nitrogen reaction gives agreement with the observed energy production of 2 ergs/g sec. All the reactions with lighter elements would give energy productions which are too large by many orders of magnitude if they were abundant enough, whereas the next heavier element, O^{16}, already gives more than 10,000 times too small a value. In view of the extremely strong dependence on the atomic number, the agreement of the nitrogen-carbon cycle with observation is excellent.

The nitrogen-carbon reactions also explain correctly the dependence of mass on luminosity for main sequence stars. In this connection, the strong dependence of the reaction rate on t

massive stars ha

slightly higher c

$T=3.2\times10^7$ and

With the assu

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Cornell Universit

Ithaca, New Yo

December 15,

TABLE I. *Energy production in the sun for several nuclear reactions.*

REACTION	AVERAGE ENERGY PRODUCTION ϵ(erg/g sec.)
$H^1+H^1=H^2+\epsilon^++f.$*	0.2
$H^2+H^1=He^3$	3×10^{18}
$Li^7+H^1=2He^4$	4×10^{16}
$B^{10}+H^1=C^{11}+f.$	3×10^8
$B^{11}+H^1=3He^4$	10^{16}
$N^{14}+H^1=O^{15}+f.$	3
$O^{16}+H^1=F^{17}+f.$	10^{-4}

* "+f." means that the energy production in the reactions following the one listed, is included. E.g. the figure for the $N^{14}+H^1$ includes the complete chain (1).

---

[1] C. F. v. Weizsaec

[2] J. Oppenheimer a

[3] G. Gamow, Phys.

[4] B. Strömgren, Er

[5] H. Bethe and C.

[6] Only for very c become important.

[7] The author is ind

1 stranica ⇨
1 Nobelova nagrada

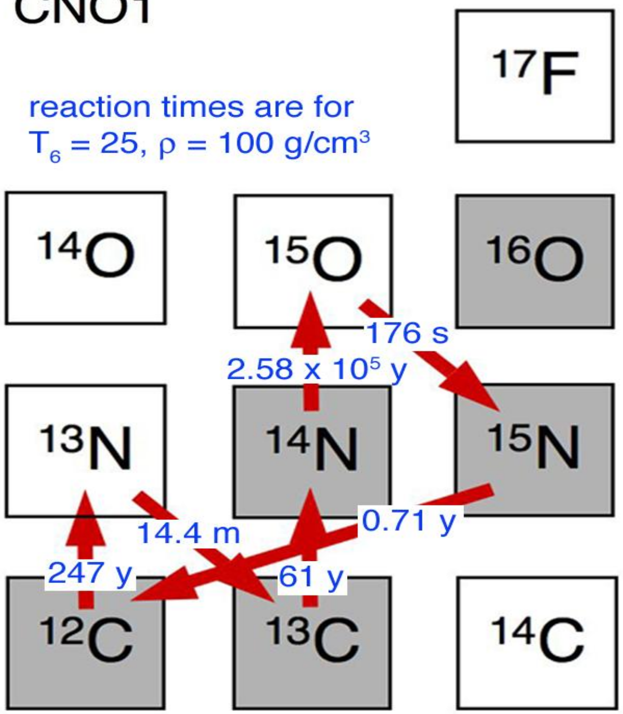
...for his contributions to the theory of nuclear reactions, especially his discoveries concerning energy production in stars.

The most important source of stellar energy appears to be the reaction cycle:

$$\begin{aligned} &C^{12}+H^1=N^{13}\ (a),\quad N^{13}=C^{13}+\epsilon^+\ (b)\\ &C^{13}+H^1=N^{14}\ (c)\\ &N^{14}+H^1=O^{15}\ (d),\quad O^{15}=N^{15}+\epsilon^+\ (e)\\ &N^{15}+H^1=C^{12}+He^4\ (f). \end{aligned} \tag{1}$$

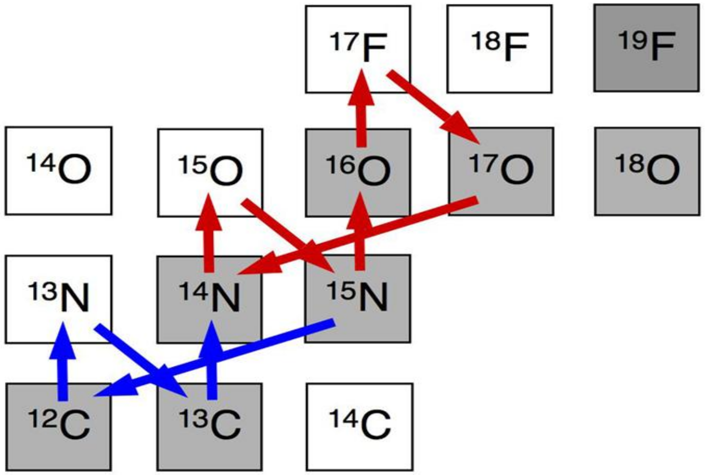
CNO1

reaction times are for $T_6 = 25$, $\rho = 100\ g/cm^3$



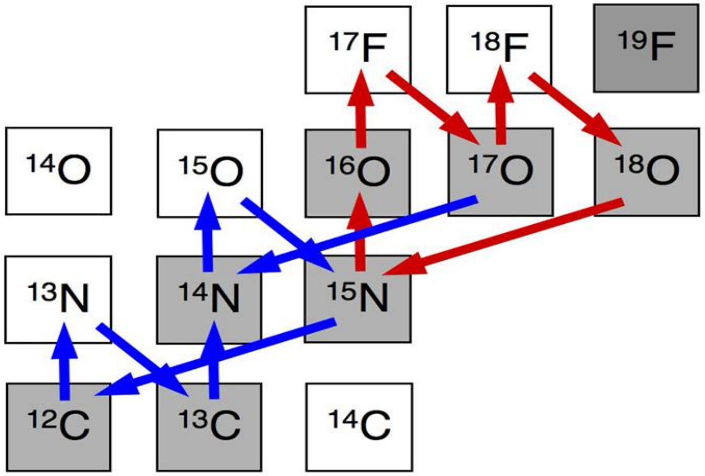
here is where the energy is generated

CNO2

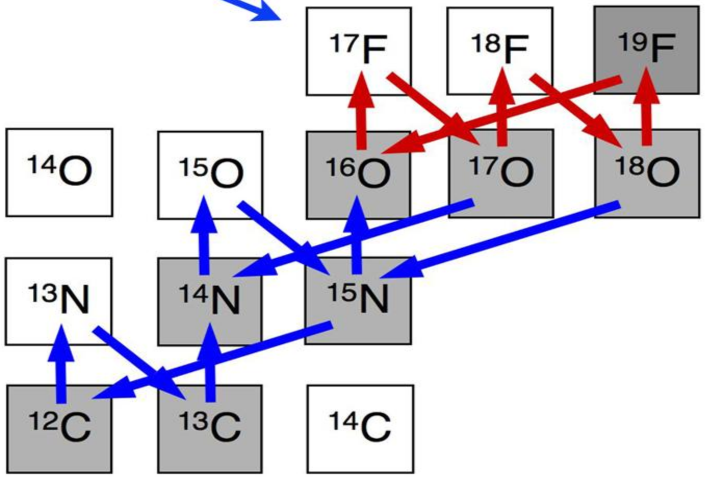


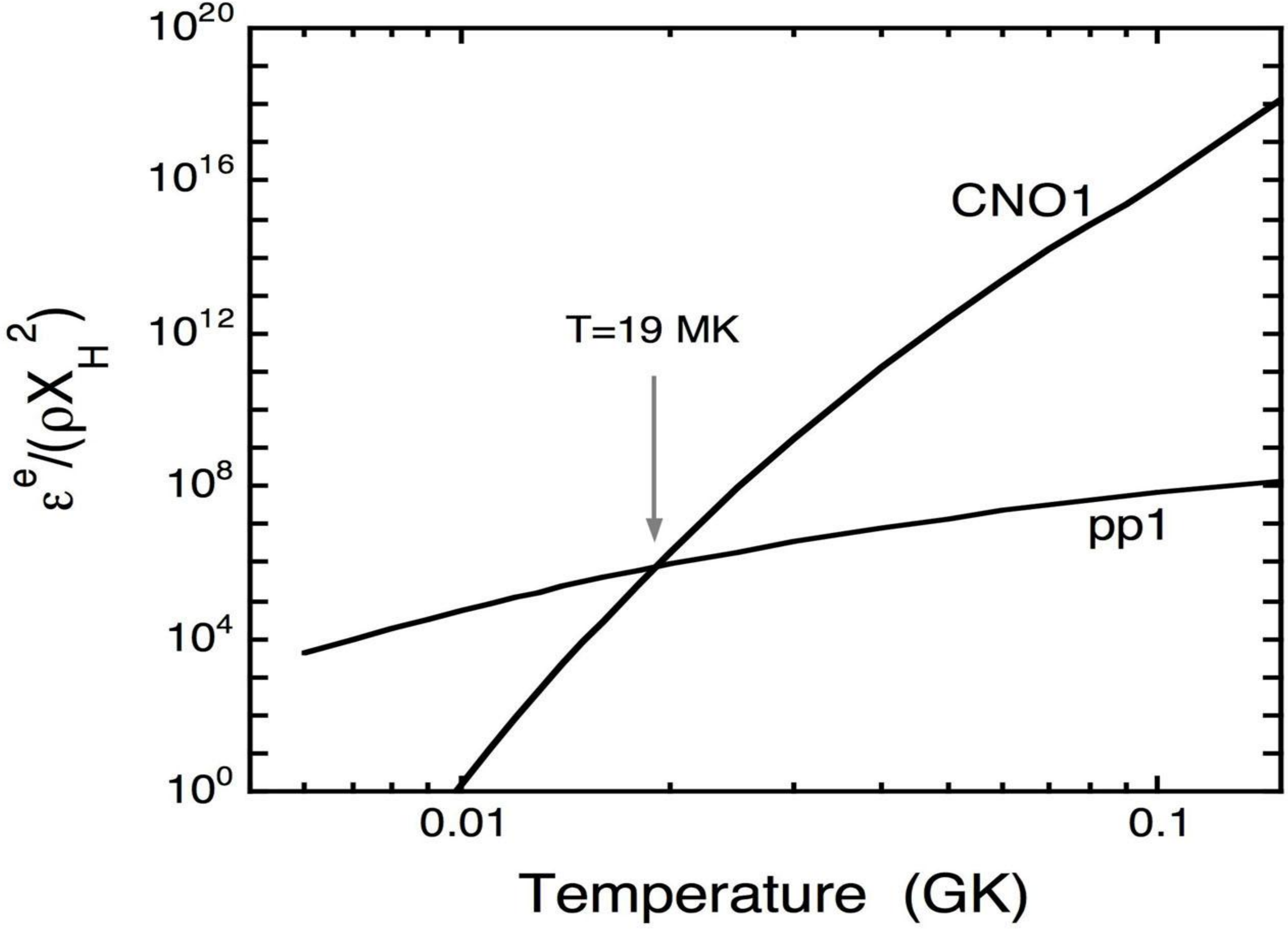
these cycles are still interesting from nucleosynthesis standpoint e.g. O isotopes can be used to probe mixing in AGB stars

CNO3



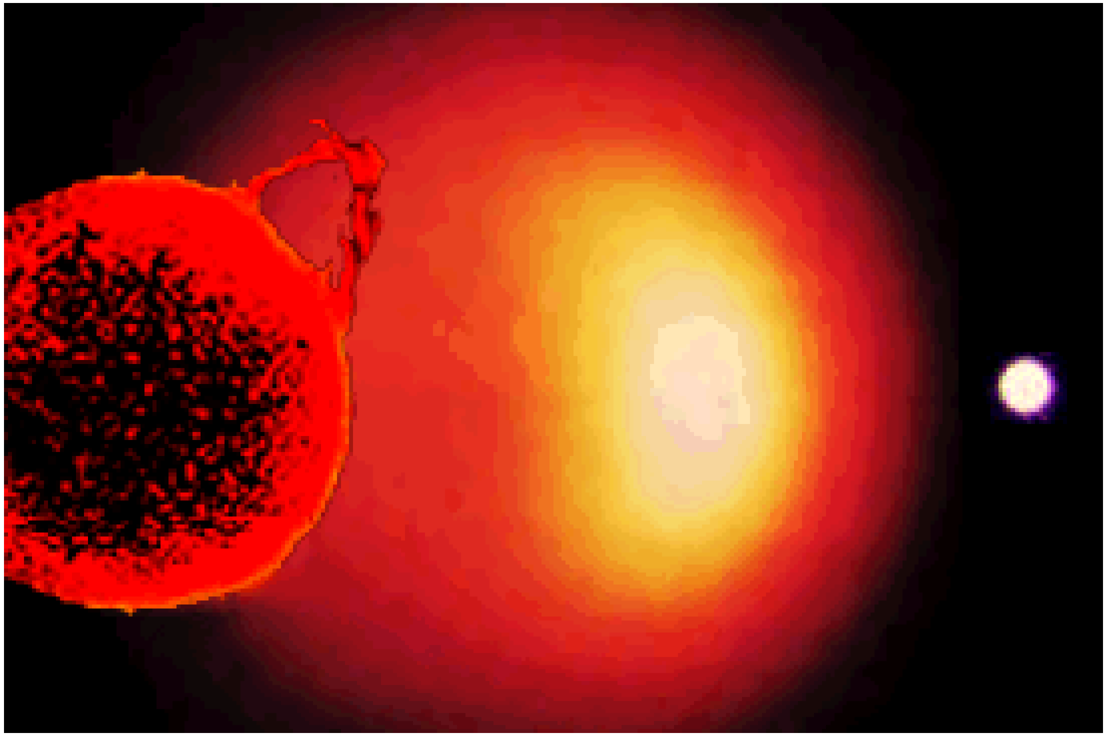
CNO4




10^{20}
10^{16}
10^{12}
10^{8}
10^{4}
10^{0}
$\varepsilon^e/(\rho X_H^2)$
CNO1
T=19 MK
pp1
0.01
0.1
Temperature (GK)

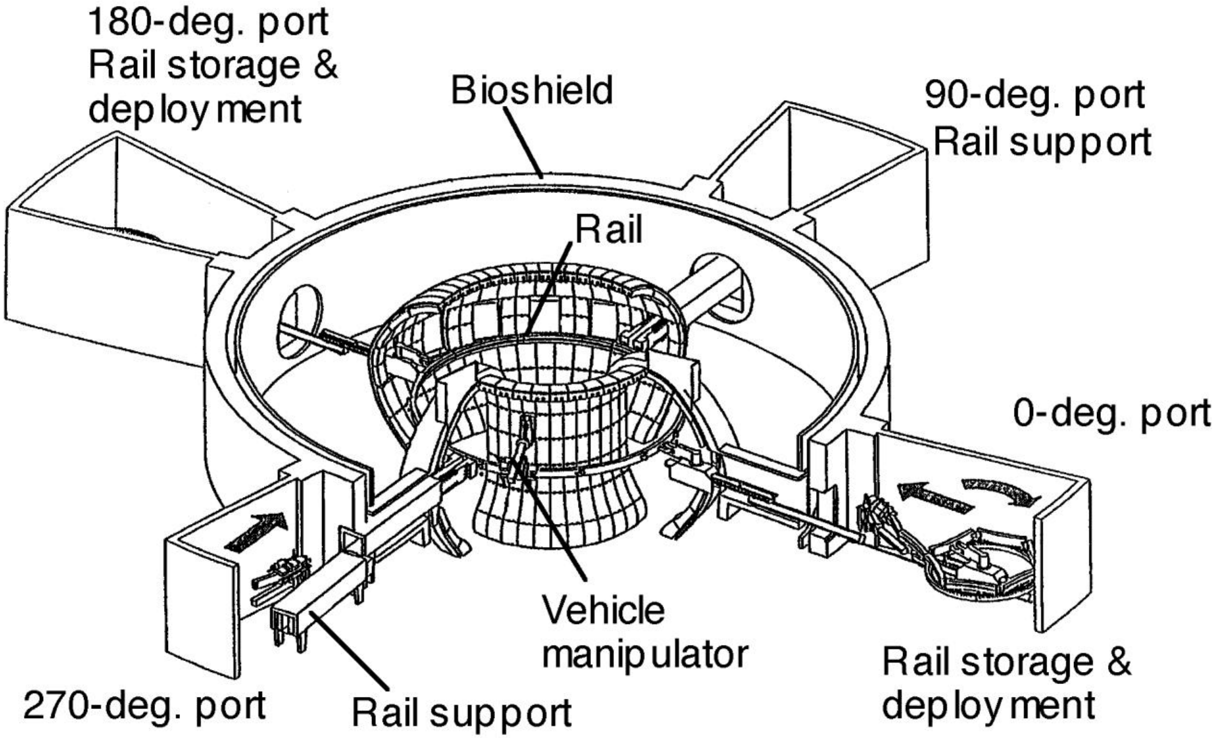
Sunce

- naše Sunce 99% energije proizvodi *pp*-ciklusom, a samo 1% kroz CNO-ciklus (za sada, taj postotak raste s vremenom)
- svake sekunde se 700 000 000 tona vodika pretvori u helij
- trenutačni sastav (maseni udio): 70% vodika, 28% helija i 2% "metala"
- starost: 4.5 milijardi godina
- kad istroši gorivo (nakon otprilike daljnjih 5 milijardi godina), Suncu će volumen prvo rasti ("crveni div"), da bi se zatim bitno smanjio ("bijeli patuljak")



ITER (Cadarache, Marseille, Francuska)

- *lat.* = put, od *engl.* International Thermonuclear Experimental Reactor
- komercijalni fuzijski reaktor?
- planirani proračun: oko 20 milijardi USD
- problem: visoka temperatura, zakočno zračenje



Gorenje helija (reakcije 3 alfa-čestice)

- ne postoje stabilne jezgre s A=5 i A=8!
- stvaranje elemenata težih od helija odvija se u dva koraka:

Prvi korak: $\alpha + \alpha \rightarrow {}^{8}\mathrm{Be}$

nestabilan: $T_{1/2} \approx 10^{-16}$ s no, malena ravnotežna količina postoji!

Drugi korak:

${}^{8}\mathrm{Be} + \alpha \rightarrow {}^{12}C^{*}$, stvorene jezgre ${}^{12}C$ pobuđene na ≈7.7 MeV, manji dio emisijom gama-zraka prelazi u osnovno stanje

- 1954. Fred Hoyle uočava da se količina ugljika u Svemiru može objasniti jedino bitnim "ubrzavanjem" (faktor 10^{5}!) drugog koraka ⇨ predviđa postojanje stanja (rezonance) na ≈7.7 MeV u ${}^{12}C$
- 1957. Cook, Fowler i Lauritsen (Caltech) eksperimentalno pronalaze to stanje (na 7.654 MeV) s točno predviđenim karakteristikama

(Fowler, Nobelova nagrada 1983.)

Zadatak 14. Gorenje helija odvija se preko čestično-nestabilne jezgre ^{8}Be i rezonancije u ^{12}C na temperaturama većim od 2×10^8 K. Procijenite ekscitacijsku energiju i svojstva te rezonancije!

Rješenje 14.

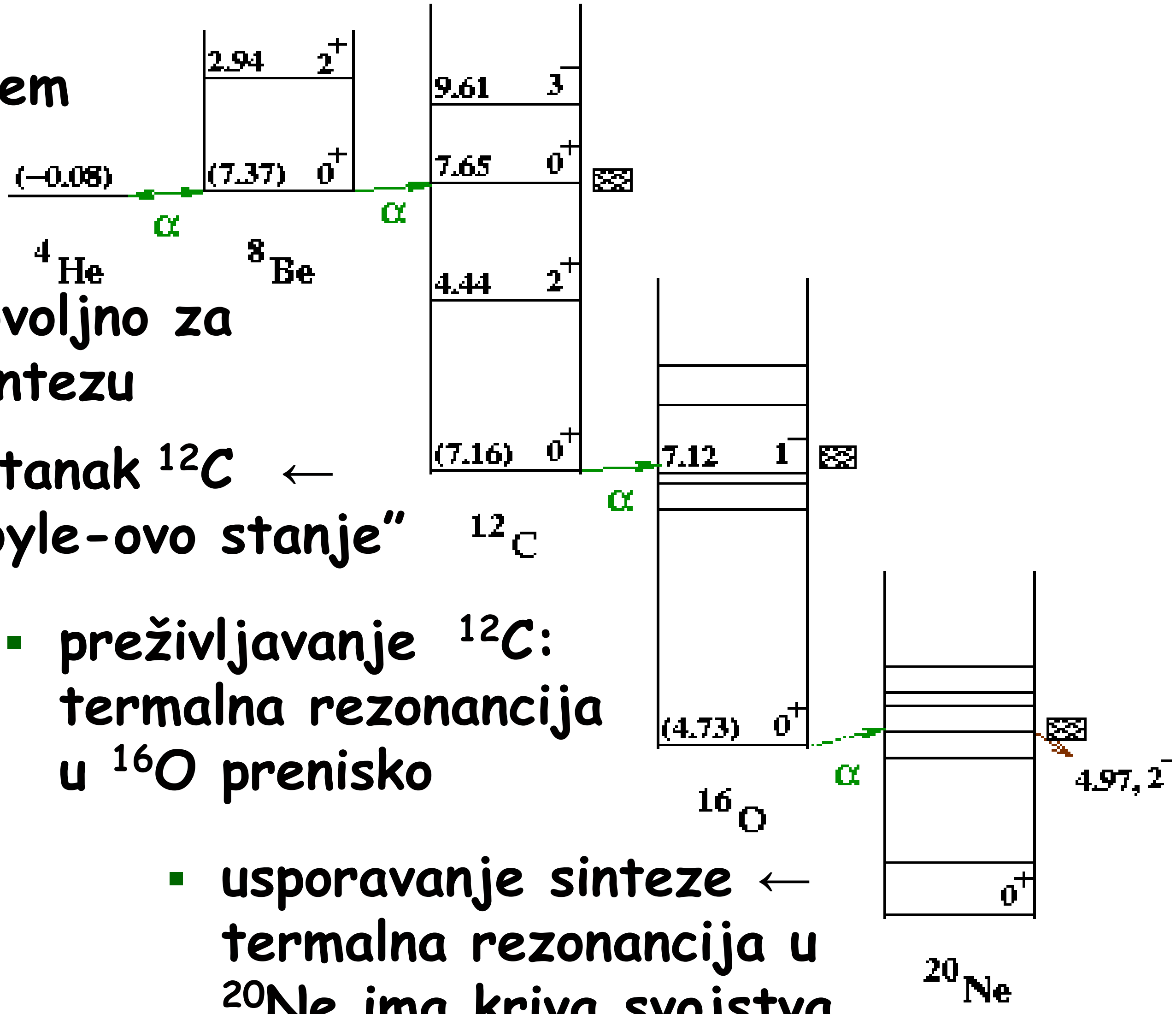
Zadatak 14. Gorenje helija odvija se preko čestično-nestabilne jezgre ^{8}Be i rezonancije u ^{12}C na temperaturama većim od 2×10^{8} K. Procijenite ekscitacijsku energiju i svojstva te rezonancije!

Rješenje 14.

- za $Z_1=2$, $Z_2=4$, $A=8/3$, $T=2\times10^{8}$ K ⇨ $E_0= 185$ keV
- slično: $\Delta E_0= 145$ keV
- dakle, rezonancija u ^{12}C bi trebala biti oko 200 keV-a iznad praga za α-raspad jezgre ^{12}C
- budući da ^{8}Be i ^{4}He imaju osnovno stanje $J^{\pi}=0^{+}$, rezonancija bi također morala imati $J^{\pi}=0^{+}$ jer svaki dodatni L podiže barijeru centrifugalnim članom...
- slično je predviđanje na temelju količine ugljika u Svemiru napravio Hoyle 1952. godine i ta je rezonancija eksperimentalno nađena par godina poslije (Cook, Fowler, Lauritsen)

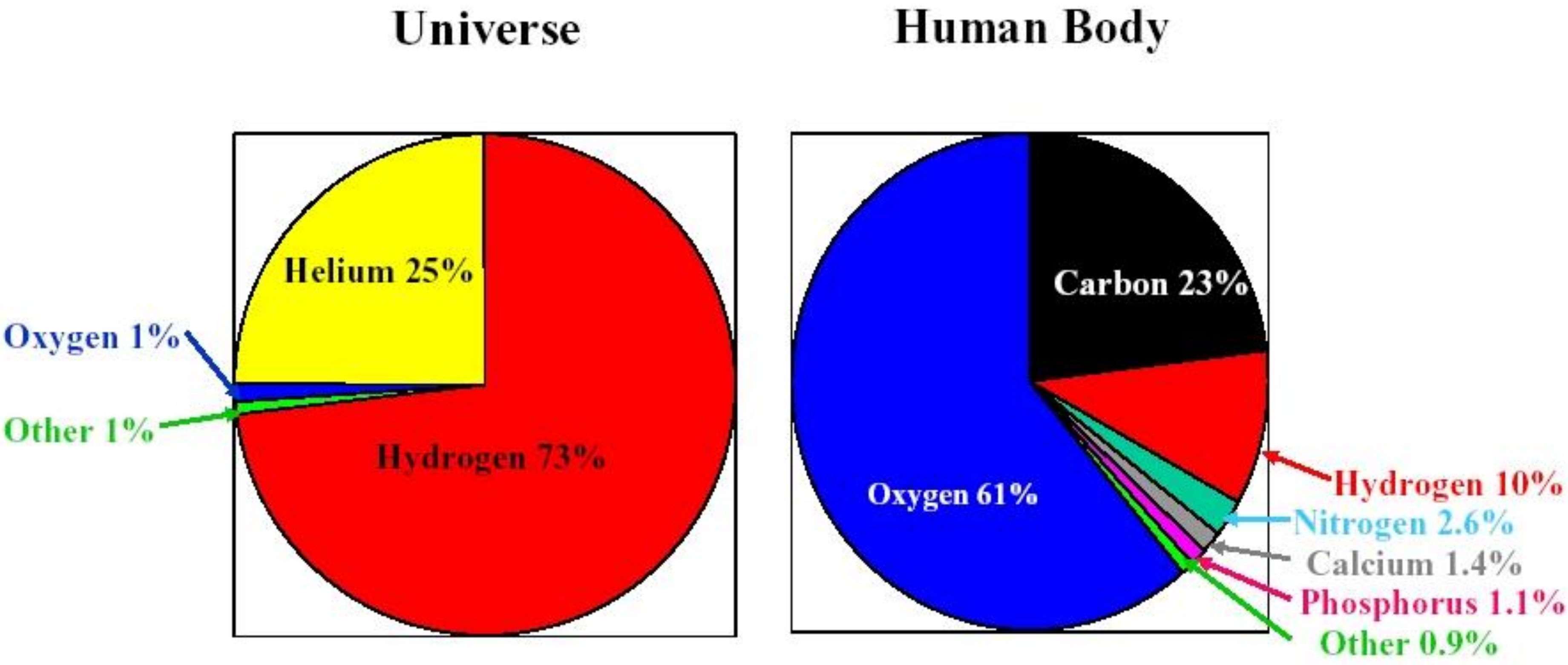
Gorenje helija, ugljika i kisika

- sinteza se nastavlja sukcesivnim dodavanjem α-čestica
  - ^{8}Be - dovoljno za daljnju sintezu
    - nastanak ^{12}C ← "Hoyle-ovo stanje"
      - preživljavanje ^{12}C: termalna rezonancija u ^{16}O prenisko
        - usporavanje sinteze ← termalna rezonancija u ^{20}Ne ima kriva svojstva

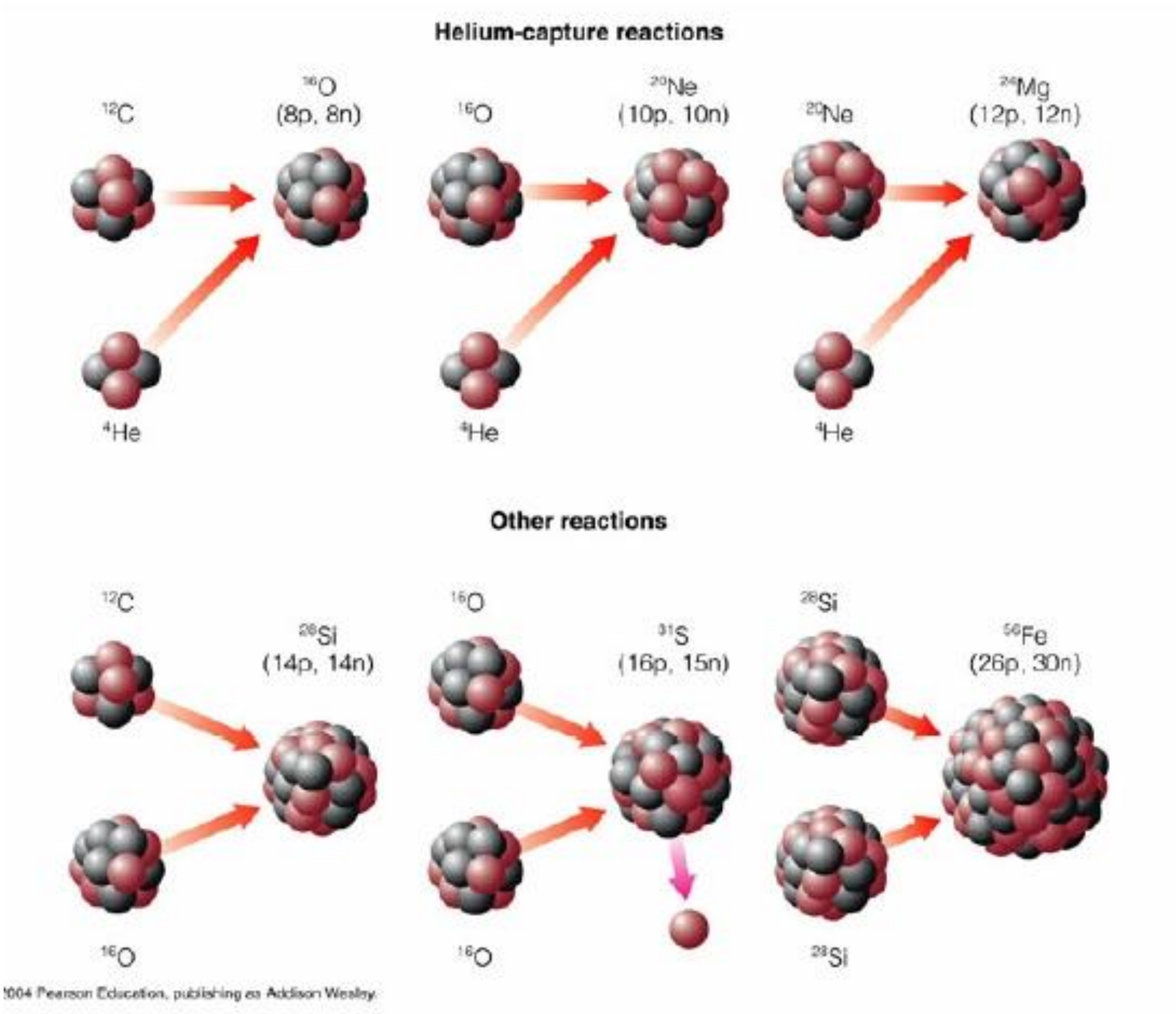


- Rezultat: puno ugljika i kisika

Ugljik i kisik



Gorenje težih jezgara



2004 Pearson Education, publishing as Addison Wesley.

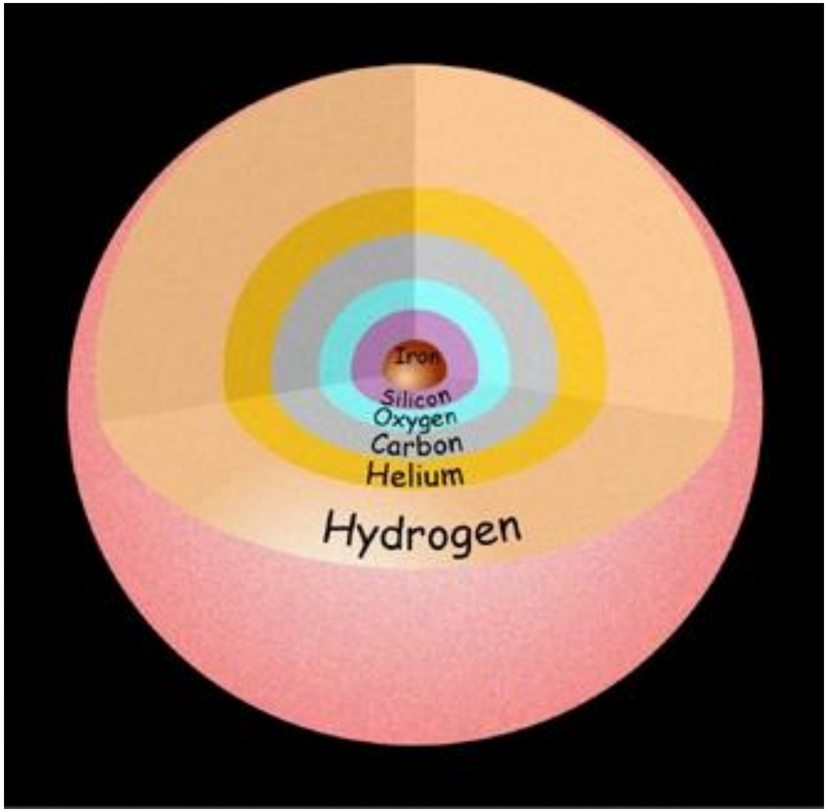
Kraj nukleosinteze u zvijezdama

- željezo je najjače vezana jezgra, pa s njom sinteza prestaje

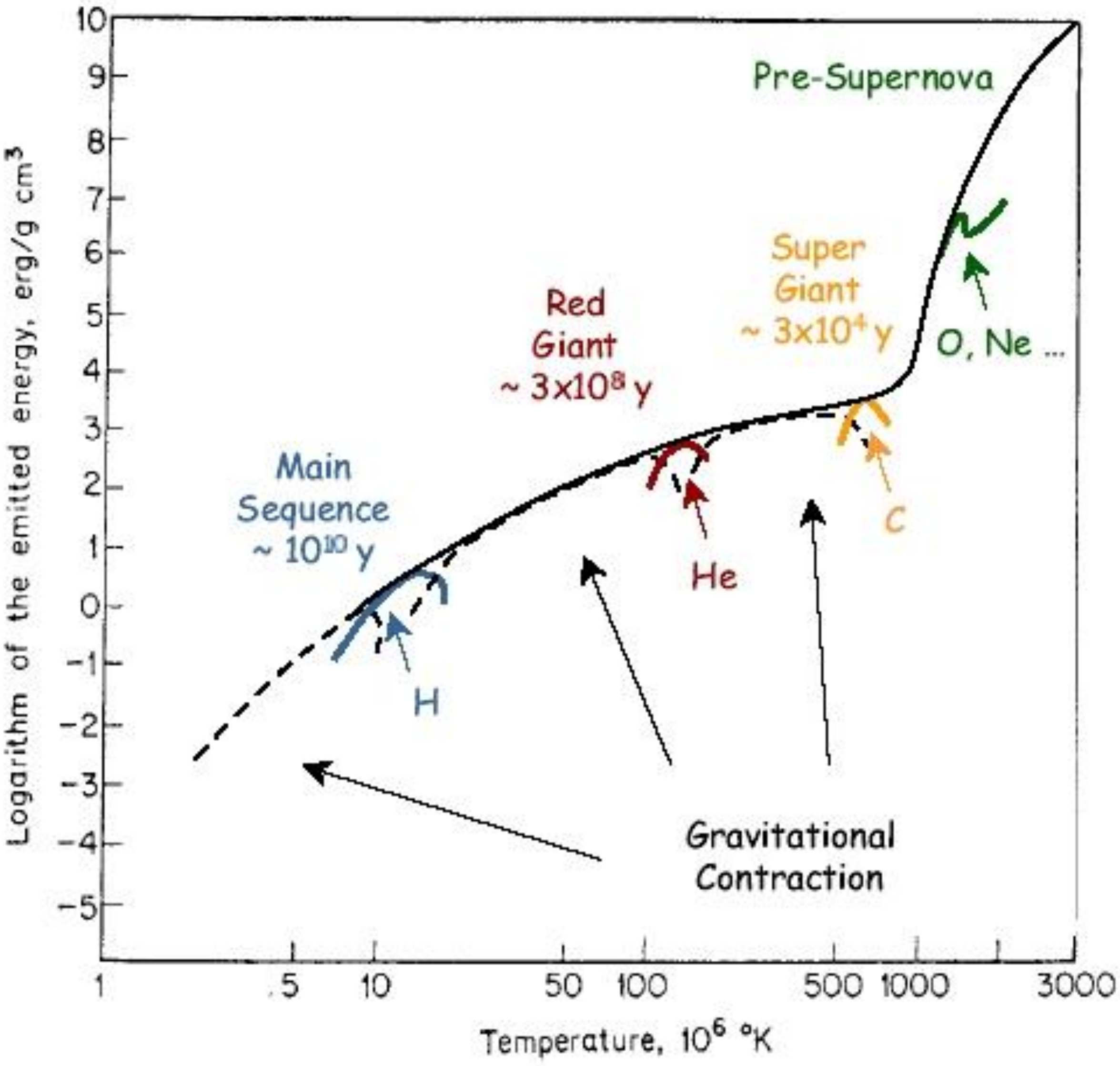
- unutrašnjost zvijezde podijeljena je na ljuske različitog sastava

- u crvenim divovima (starim zvijezdama velikog volumena) sinteza elemenata se nastavlja tzv. *s-procesima* (sporim procesima)

- neutroni stvoreni u vanjskim ljuskama uzrokuju reakcije na željezu, stvorene jezgre raspadaju se beta-raspadom i tako nastaje manja količina težih elemenata



Daljnje gorenje težih izotopa



Kraj gorenja

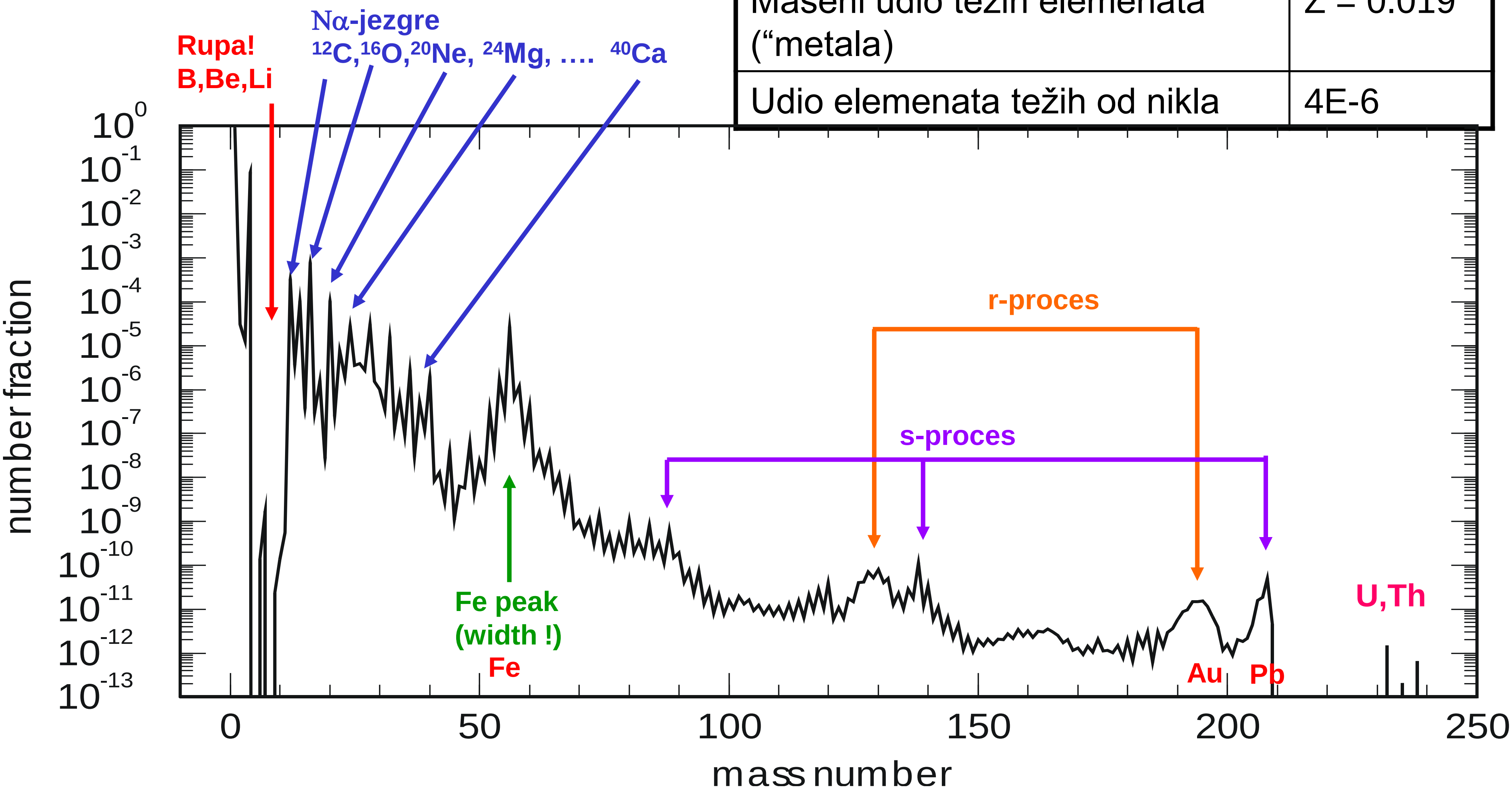
Masa ($M_\odot$)	Konačan stupanj
< 0.08	nema fuzije!
0.1 -0.5	gorenje vodika
0.5 - 8	gorenje helija
8 - 11	gorenje ugljika
> 11	Supernova

Evolucija zvijezde s masom 25 $M_\odot$

	Vrijeme	T_c (10^9 K)	ρ (g cm^{-3})
gorenje vodika	7×10^6 god	0.06	5
gorenje helija	5×10^5 god	0.23	7×10^2
C/O - gorenje	600 / 0.5 god	0.93 – 2.3	$2 \times 10^5 - 1 \times 10^7$
gorenje Si	1 dan	4.1	3×10^7
eksplozivni procesi	0.1 – 1 s	1.2 - 7	

Zastupljenost elemenata

Maseni udio vodika	X = 0.71
Maseni udio helija	Y = 0.28
Maseni udio težih elemenata (“metala)	Z = 0.019
Udio elemenata težih od nikla	4E-6

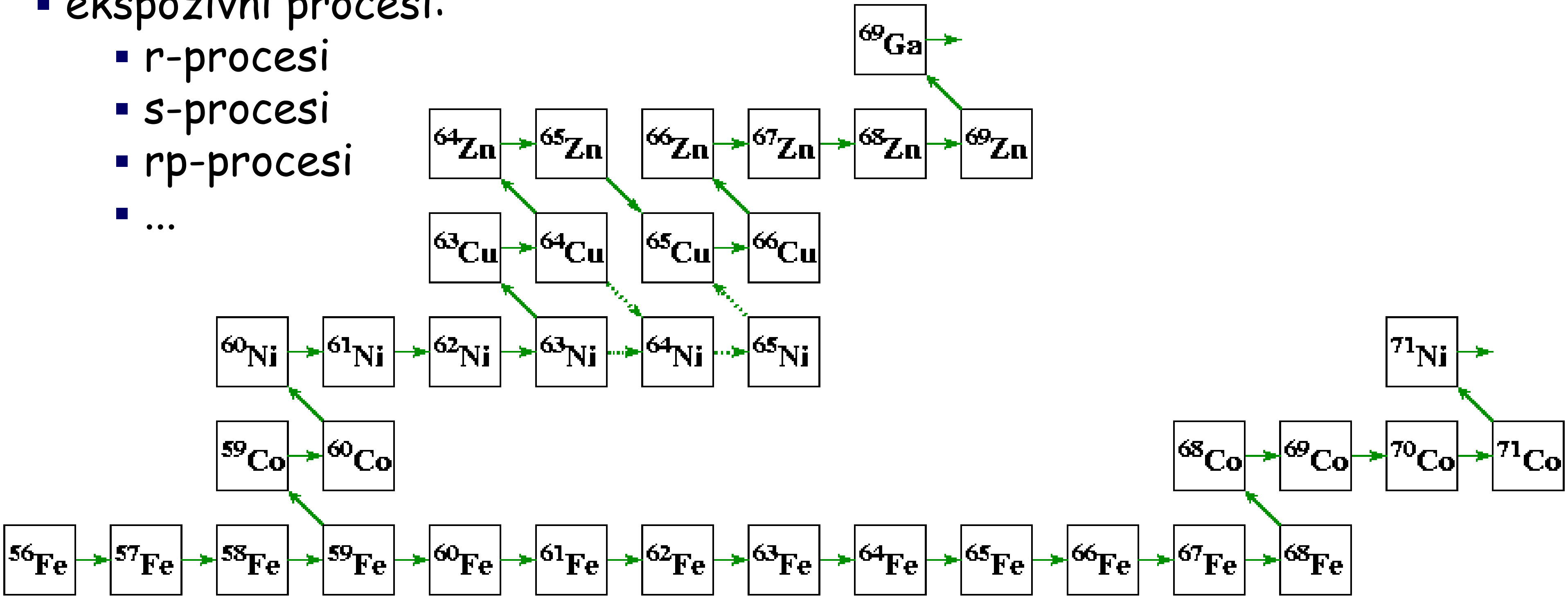


Supernove

- stvorene jezgre raspršuju se Svemirom u eksplozijama supernova

- pri tome nastaju i elementi teži od željeza (uglavnom uhvatom neutrona kombiniranim s beta-raspadom)...

- ekspozivni procesi:
    - r-procesi
    - s-procesi
    - rp-procesi
    - ...



Još jedan dokaz nukleosinteze u zvijezdama

- u zvijedama kod kojih se (za razliku od našeg Sunca) materijal iz unutrašnjosti prenosi do fotosfere, uočene (1952.) spektralne linije tehnicija (Tc)
- tehnicij nema stabilnih izotopa – najdugovječniji ^{98}Tc ima vrijeme poluživota od ≈ 4 milijuna godina

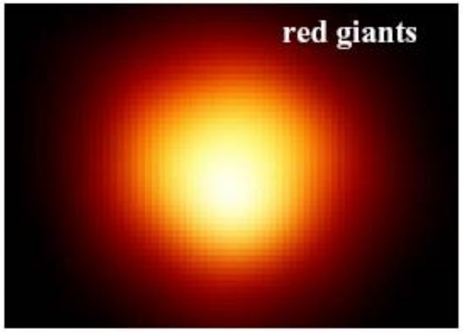
dokaz nukleosinteze u zvijezdama

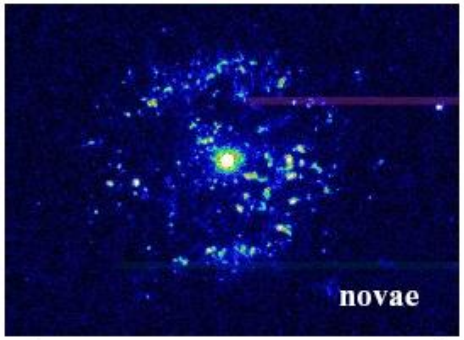
Vremenska skala

Main sequence star
$T_6 \sim 15$, $M_\odot \sim 1$,
$t \sim 10^{10}$ y

Red Giant star;
$T_9 \sim 0.1 - 0.2$,
$M_\odot \sim 1$, $t \sim 10^9$ y

$T_9 \sim 0.1$,
$M_\odot \sim 10^2 - 10^5$
$t < 10^6$ y

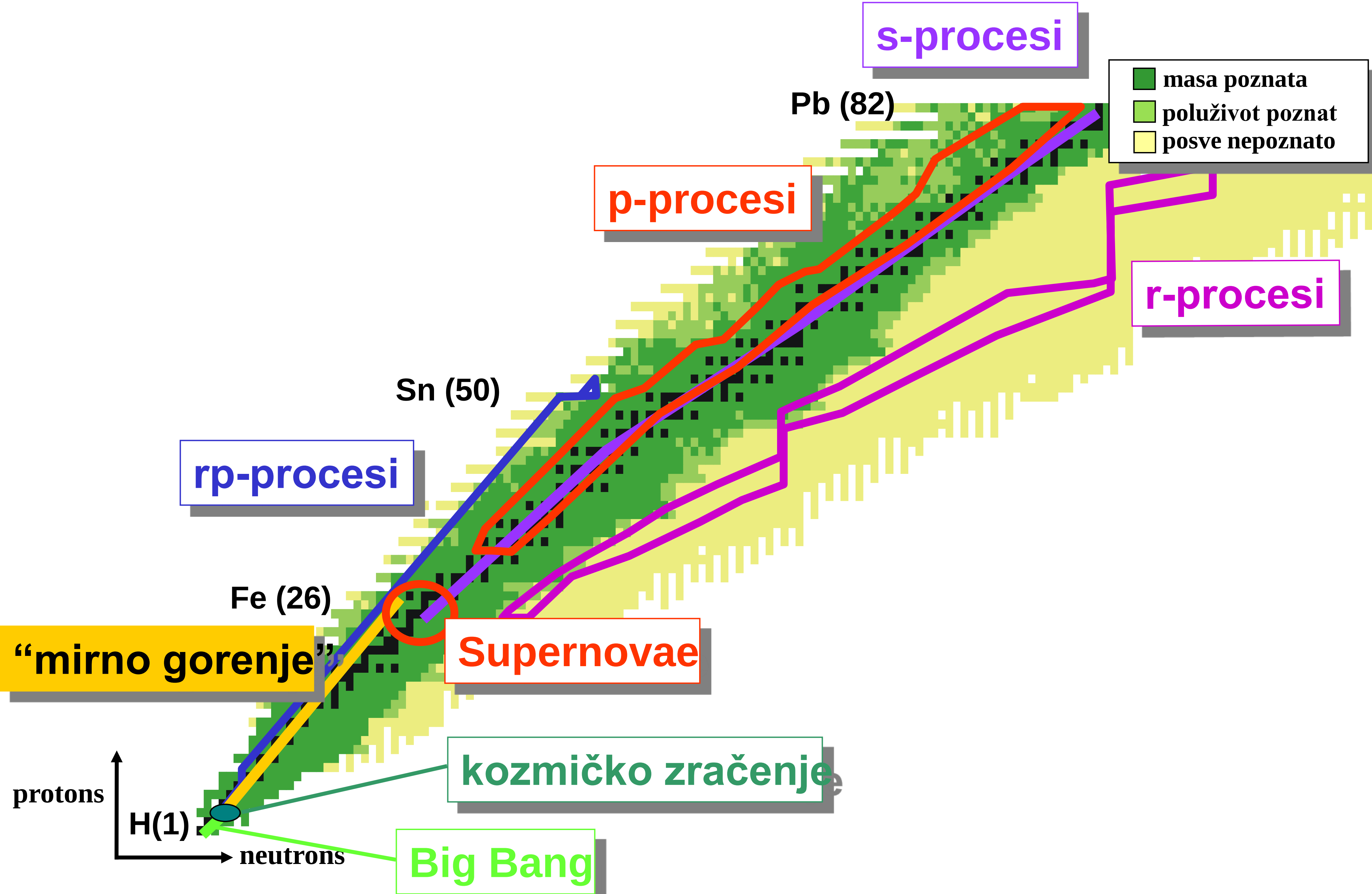




White dwarf star
$T_9 \sim 0.1 - 0.4$, $M_\odot \sim 1$,
$t \sim$ days

Neutron star
$T_9 \sim 0.7 - 2$, $M_\odot \sim 1$,
$t \sim 10$ sec

massive star
$T_9 \sim 1$, $M_\odot \sim 10$,
$t \sim$ sec



s-procesi

🟩 masa poznata
🟩 poluživot poznat
🟨 posve nepoznato

Pb (82)

p-procesi

r-procesi

Sn (50)

rp-procesi

Fe (26)

“mirno gorenje”

Supernovae

kozmičko zračenje

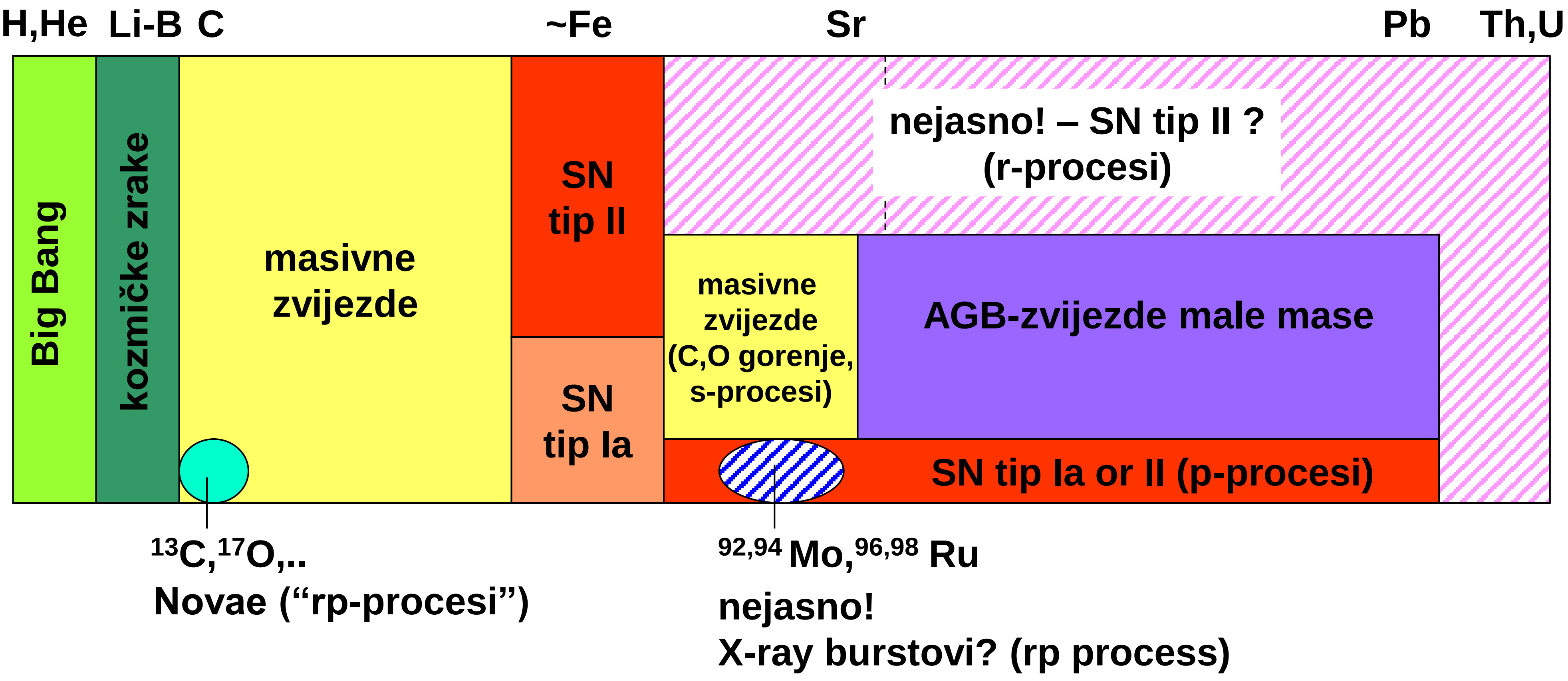
protons

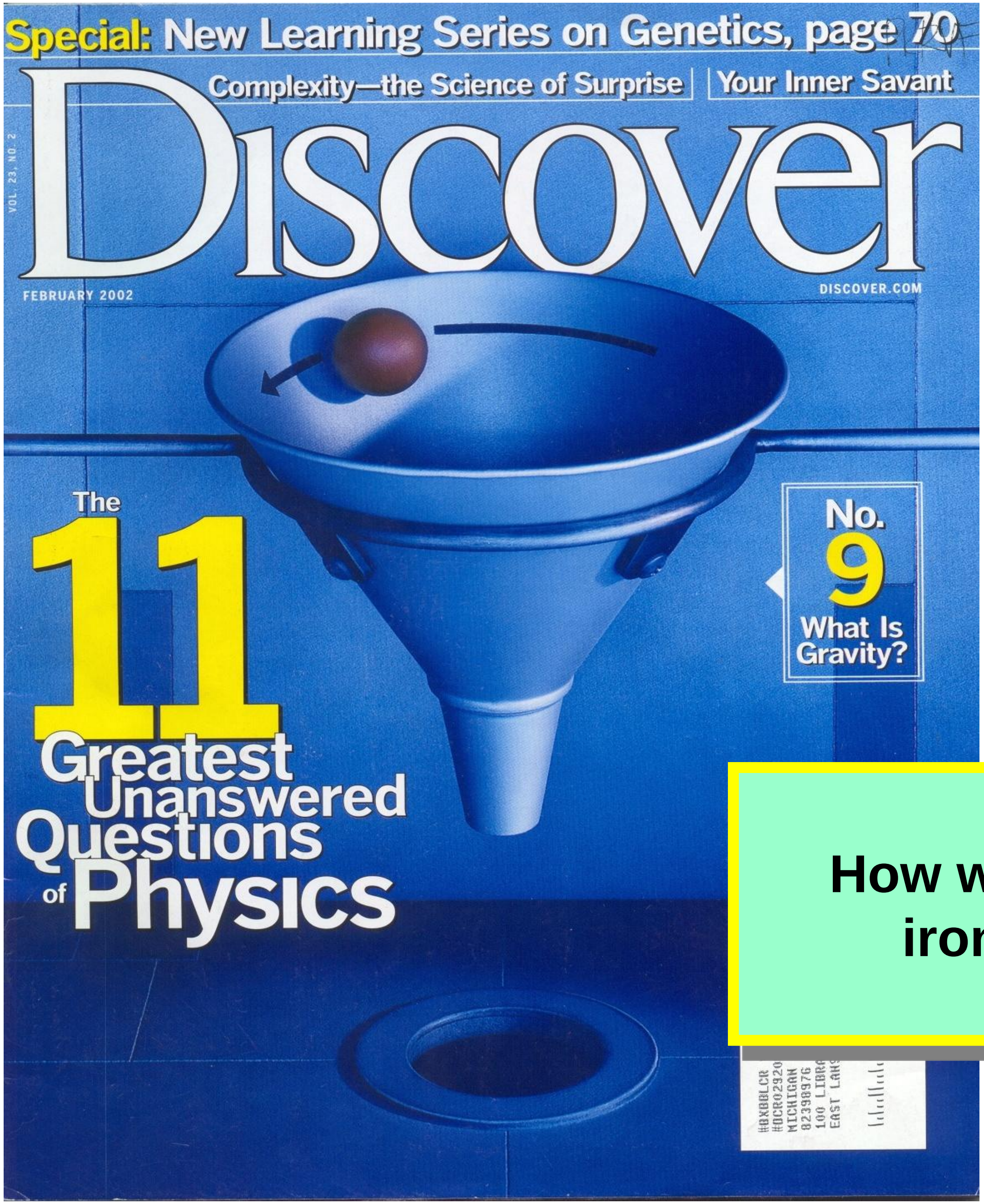
H(1)

neutrons

Big Bang

Porijeklo elemenata



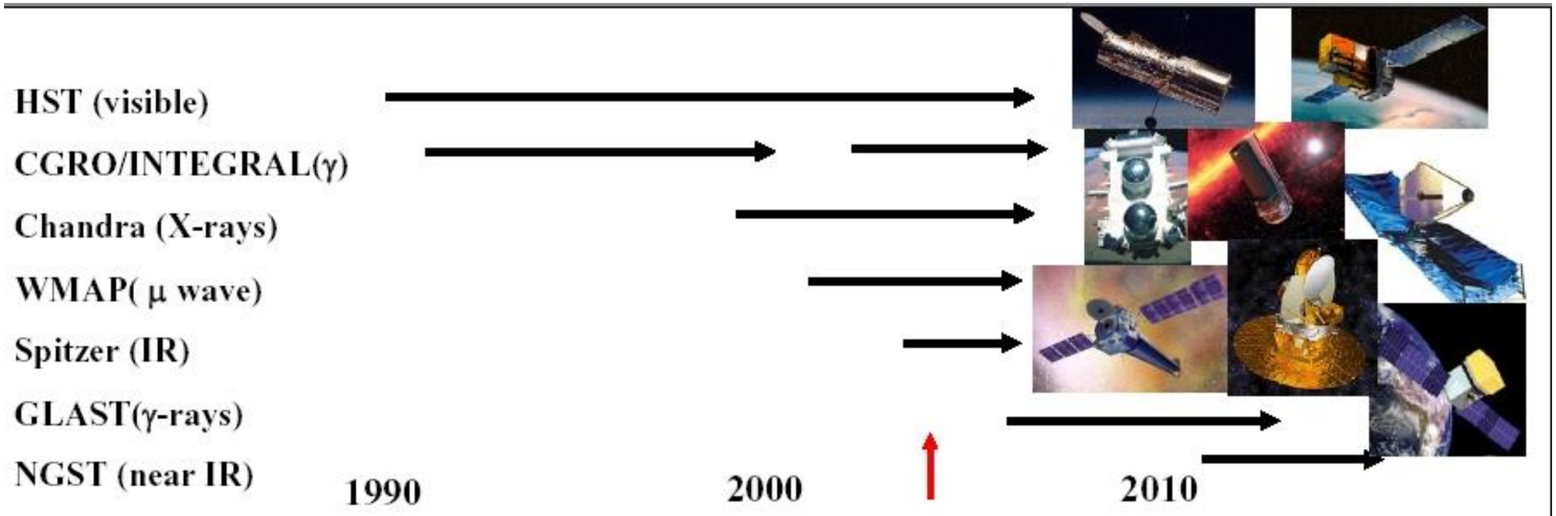


Based on National Academy of Science Report

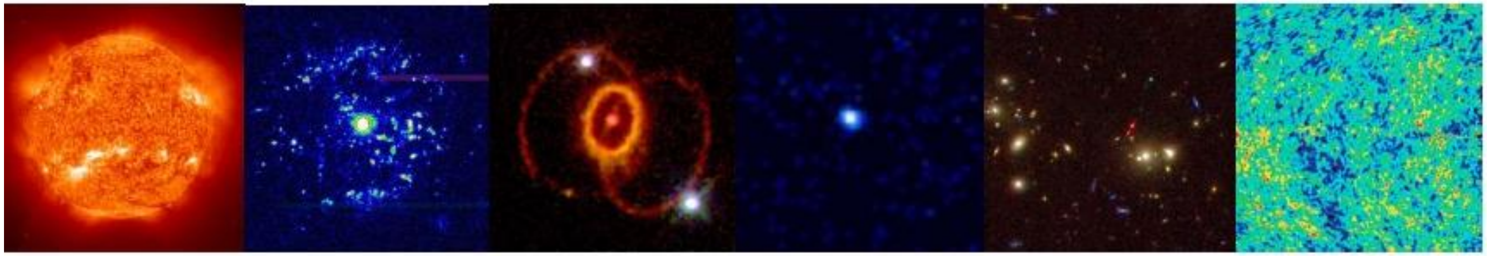
[Committee for the Physics of the Universe (CPU)]

Question 3
How were the elements from iron to uranium made ?

Post-festum


HST (visible)
CGRO/INTEGRAL(γ)
Chandra (X-rays)
WMAP(μ wave)
Spitzer (IR)
GLAST(γ-rays)
NGST (near IR)

Main sequence star
Novae
Supernovae
Neutron star
Population III stars
Big Bang



Neutrino oscillations
Synthesis of the Elements
Accelerating universe
Dense Matter
Supermassive stars
Cosmology
Dark energy

Zadatak 15. **Izvedite izraz koji opisuje ovisnost tlaka i gustoće materije za bijele patuljke, te pomoću tog izraza riješite jednadžbe strukture za te zvijezde.**

Rješenje 15. (početak izvoda jednak za sve zvijezde)

- pretpostavljamo sferičnu simetriju
- prva jednadžba:

$$\boxed{\frac{dM(r)}{dr} = 4\pi r^2 \rho(r)}$$

- druga jednadžba (balans sila na *d* M):

$$dF_{tot} = dF_{grav} + dF_{tlak}$$

$$dF_{grav} = -\frac{GM(r)dM}{r^2}$$

Rješenje 15.

- dalje vrijedi:

$$dF_{tlak} = \left[P(r) - P(r+dr)\right] dS$$

$$dM = \rho dS dr$$

$$dF_{tot} = \ddot{r} dM$$

- ⇨

$$\frac{dP}{dr} = -\frac{GM\rho}{r^2} - \rho \frac{d^2 r}{dt^2}$$

- u ravnoteži:

$$\boxed{\frac{dp(r)}{dr} = -\frac{GM(r)\rho(r)}{r^2}}$$

- dobili smo dvije jednadžbe s tri nepoznanice (M, ρ, P). Treba nam još jedna – "jednadžba stanja".

Rješenje 15.

- općenito, jednadžba stanja može uključivati nove varijable (temperaturu, sastav materijala, proizvodnju energije, ...) što vodi na potrebu za novim jednadžbama da bi do kraja definirali sistem
- za vrlo specijalan slučaj kada postoji jednostavna (politropska) relacija između tlaka i gustoće:

$$\boxed{P = K\rho^{\Gamma} = K\rho^{1+1/n}}$$

sistem jednadžba se može riješiti sljedećom procedurom:

- specificiramo $P(0)$ ili $r(0)$;
- integriramo prema van;
- u točki $r=R$ zahtijevamo $P(R)=0$.

Rješenje 15.

- imamo:

$$\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dP}{dr}\right)=\frac{1}{r^2}\frac{d}{dr}\left(-GM(r)\right)=$$

$$=\frac{1}{r^2}\left(-G\frac{dM(r)}{dr}\right)=$$

$$=-\frac{1}{r^2}G4\pi r^2\rho=$$

$$=-4\pi G\rho \qquad \textbf{(*)}$$

- ansatz:

$$\rho(r)\equiv\rho(0)\Phi(\xi)^n \qquad \xi\equiv r/a$$

$$a\equiv\left[\frac{(n+1)K\rho(0)^{(1-n)/n}}{4\pi G}\right]^{1/2}$$

Rješenje 15.

- rubni uvjeti postaju:

$$\Phi(0)=1$$
$$\Phi\left(R/a\right)=0$$

- a uvrštavanjem u (*) dobiva se:

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\Phi}{d\xi}\right)=-\Phi^n$$

- to je tzv. Lane-Emdenova jednadžba i njena su rješenja tabelirana za neke vrijednosti *n* -a (ne nužno cjelobrojnog)
- rješenje za masu dano je s:

$$M(R)=-4\pi\left[\frac{(n+1)K}{4\pi G}\right]^{3/2}\rho^{(3-2n)/2n}\left(\xi^2\frac{d\Phi}{d\xi}\right)_{\xi=\xi_1=R/a}$$

n	ξ_1	ρ_0	P_c	γ	T_0	U
		$\frac{M}{R^3}$	$\frac{GM^2}{R^4}$	$\frac{1}{MR^2}$	$\frac{GM\mu}{kR}$	$-\frac{GM^2}{R}$
0	$\sqrt{6}$	$\frac{15}{8\pi}$	$\frac{15}{16\pi}$	$\frac{2}{7}$	$\frac{1}{2}$	$\frac{5}{14}$
1	π	$\frac{\pi}{4}$	$\frac{\pi}{8}$	$\frac{2(\pi^2-6)}{3\pi^2}$	$\frac{1}{2}$	$\frac{3}{8}$
$\frac{3}{2}$	3.653					
2	4.353	1.12	0.55	0.24	0.49	0.40
$\frac{5}{2}$	5.355					
3	6.896	1.84	0.83	0.22	0.45	0.42
$\frac{7}{2}$	9.535					
4	14.97	4.16	1.48	0.21	0.36	0.44
$\frac{9}{2}$	31.83					
5	∞					

Rješenje 15.

- trošenjem goriva zvijezda se hladi i sažima; na dovoljno malenoj temperaturi elektroni popunjavaju najniža moguća stanja (do Fermijevog nivoa):

$$n = \frac{8\pi}{(2\pi\hbar)^3}\int_0^{k_F} k^2 dk = \frac{k_F^3}{3\pi^2\hbar^3}$$

- ako je μ omjer broja nukleona i elektrona, a m_N masa nukleona, vrijedi:

$$\rho = n m_N \mu$$

$$k_F = \hbar\left(\frac{3\pi^2\rho}{m_N\mu}\right)^{1/3}$$

Rješenje 15.

- tlak plina dan je s:

$$P = \frac{8\pi}{3(2\pi\hbar)^3} \int_0^{k_F} \frac{k^2}{\left(k^2 + m_e^2\right)^{1/2}} k^2 dk =$$

$$= \frac{m_e^4}{3\pi^2\hbar^3} \int_0^{k_F/m_e} \frac{u^4}{\left(1+u^2\right)^{1/2}} du =$$

$$= \frac{m_e^4}{24\pi^2\hbar^3} \left[\left(2u_0^3 - 3u_0\right)\left(1+u_0^2\right)^{1/2} + 3\sinh^{-1} u_0 \right]$$

- gdje je:

$$u_0 = k_F / m_e$$

Rješenje 15.

- točka kad je:

$$k_F = m_e$$

definira granicu između relativističke i nerelativističke materije kojoj odgovara granična gustoća:

$$\rho_c = \frac{m_N \mu}{3\pi^2}\left(\frac{m_e c^2}{\hbar c}\right)^3 = 0.97 \cdot 10^6 \mu \ \text{g/cm}^2$$

- u limesu $\rho \ll \rho_c$ (nerelativistički limes), dobiva se:

$$P = \frac{8\pi}{3(2\pi.\hbar)^3}\frac{1}{m_e}\frac{k_F^5}{5} = \frac{\hbar^2}{15\pi^2 m_e}\left(\frac{3\pi^2 \rho}{m_N \mu}\right)^{5/3}$$

Rješenje 15.

- u limesu $\rho >> \rho_c$ (ultra-relativistički limes), dobiva se:

$$P = \frac{8\pi}{3(2\pi\hbar)^3}\frac{k_F^4}{4} = \frac{\hbar}{12\pi^2}\left(\frac{3\pi^2\rho}{m_N\mu}\right)^{4/3}$$

Rješenje 15.

- degeneirarni ne-relativistički elektronski plin:

$$P = K\rho^{5/3} \quad \Rightarrow n = 1.5$$

$$M = \frac{2.72}{\mu^2}\left(\frac{\rho(0)}{\rho_c}\right)^{1/2} M_0$$

$$R = \frac{2\cdot 10^4}{\mu}\left(\frac{\rho(0)}{\rho_c}\right)^{-1/6} km$$

- vrijedi za bijele patuljke male mase

Rješenje 15.

- degeneirarni ultra-relativistički elektronski plin:

$$P = K\rho^{4/3} \quad \Rightarrow n = 3$$

$$M = \frac{5.87}{\mu^2} M_0$$

$$R = \frac{5.3 \cdot 10^4}{\mu} \left(\frac{\rho(0)}{\rho_c} \right)^{-1/3} km$$

- vrijedi za bijele patuljke velike mase
- zaključak - postoji maksimalna masa za stabilnost hladnih bijelih patuljaka i ona je jednaka:

$$M = \frac{5.87}{\mu^2} M_0$$