DIFERENCIJALNA SVOJSTVA POLJA HOR. VJETRA (DODATAK G)

- neka su u i v komponente vjetra u smjeru osi x i y i neka su te komponente funkcije tih osi:

$$u = u(x,y) \quad ; \quad v = v(x,y)$$

- tada se u i v mogu razviti u Taylorov razvoj oko točke (x_0, y_0) koja će označavati ishodište koord. sustava:

$$f(x,y) = f(x_0,y_0) + \left[\left(\frac{\partial f}{\partial x}\right)_{(x_0,y_0)}(x-x_0) + \left(\frac{\partial f}{\partial y}\right)_{(x_0,y_0)}(y-y_0)\right] +$$

$$+ \frac{1}{2}\left[\left(\frac{\partial^2 f}{\partial x^2}\right)_{(x_0,y_0)}(x-x_0)^2 + 2\left(\frac{\partial^2 f}{\partial x \partial y}\right)_{(x_0,y_0)}(x-x_0)(y-y_0) + \left(\frac{\partial^2 f}{\partial y^2}\right)_{(x_0,y_0)}(y-y_0)^2\right] + \dots$$

OPĆENITO

- sada, za $(x_0, y_0) \equiv (0,0)$:

$$u(x,y) = u_0 + \left(\frac{\partial u}{\partial x}\right)_0 x + \left(\frac{\partial u}{\partial y}\right)_0 y + \dots$$

$$v(x,y) = v_0 + \left(\frac{\partial v}{\partial x}\right)_0 x + \left(\frac{\partial v}{\partial y}\right)_0 y + \dots$$

zadržavamo se na razini linearnih promjena u bliskoj okolini ishodišta pa su viši članovi razvoja zanemareni!

- prostorne derivacije komponenti horizontalnog vjetra mogu se kombinirati na sljedeći način:

(1) $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \delta$ ... horizontalna divergencija $\Rightarrow \nabla_H \cdot \vec{V}$

(2) $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \zeta$ ... vert. komp. vrtložnosti $\Rightarrow \vec{k} \cdot (\nabla \times \vec{V})$

(3) $\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} = D_1$ ... deformacija rastezanja

(4) $\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = D_2$ ... deformacija smicanja

- koristeći gornje "supstitucije", pojedine derivacije se mogu zapisati kao:

$(1)+(3) \Rightarrow \frac{\partial u}{\partial x} = \frac{1}{2}(\delta + D_1)$

$(1)-(3) \Rightarrow \frac{\partial v}{\partial y} = \frac{1}{2}(\delta - D_1)$

$(2)+(4) \Rightarrow \frac{\partial v}{\partial x} = \frac{1}{2}(\zeta + D_2)$

$(2)-(4) \Rightarrow \frac{\partial u}{\partial y} = -\frac{1}{2}(\zeta - D_2)$

to sada u Taylorov razvoj

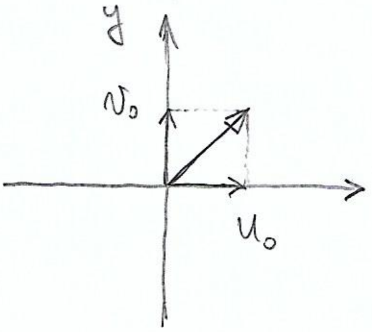
(*)

$$\Rightarrow \boxed{\begin{aligned} u(x,y) &= u_0 + \frac{1}{2}(\delta + D_1)x - \frac{1}{2}(\zeta - D_2)y = u_0 + \frac{1}{2}\delta x - \frac{1}{2}\zeta y + \frac{1}{2}D_1 x + \frac{1}{2}D_2 y \\ v(x,y) &= v_0 + \frac{1}{2}(\zeta + D_2)x + \frac{1}{2}(\delta - D_1)y = v_0 + \frac{1}{2}\delta y + \frac{1}{2}\zeta x - \frac{1}{2}D_1 y + \frac{1}{2}D_2 x \end{aligned}}$$

- specijalni slučajevi po članovima u jdžbama za brzine

① $\delta, \zeta, D_1, D_2 = 0 \;;\; u_0, v_0 \neq 0$

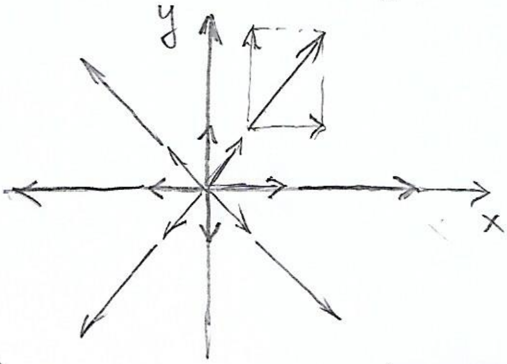
$\Rightarrow u = u_0 \;;\; v = v_0$ i neka su $u_0, v_0 > 0$



$\Rightarrow$ TRANSLACIJA

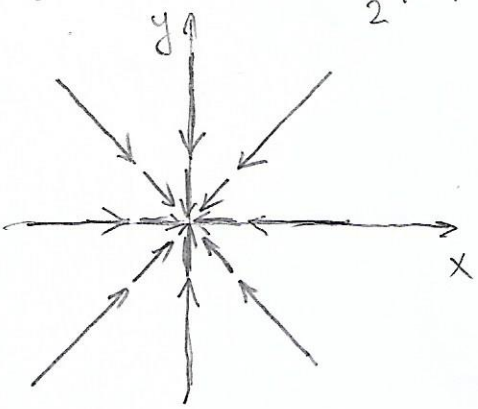
② $\zeta, D_1, D_2, u_0, v_0 = 0 \;;\; \delta \neq 0$

2a) $\delta > 0 \Rightarrow u = \frac{1}{2}\delta x \;;\; v = \frac{1}{2}\delta y \Rightarrow$ porastom x i y rastu u i v



$\Rightarrow$ DIVERGENCIJA $\Rightarrow$ brzina se povećava udaljavanjem od ishodišta

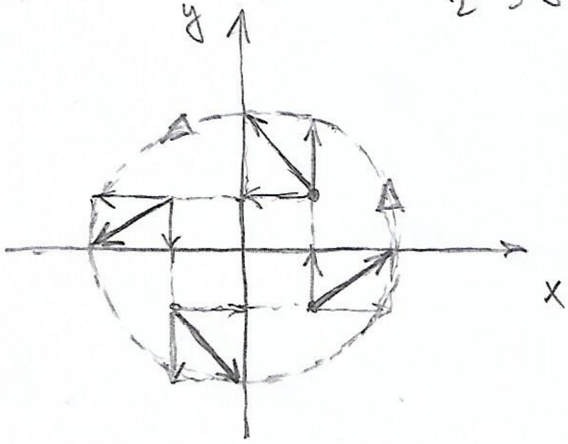
2b) $\delta < 0 \Rightarrow u = -\frac{1}{2}|\delta| x \;;\; v = -\frac{1}{2}|\delta| y$



$\Rightarrow$ KONVERGENCIJA $\Rightarrow$ udaljavanjem od ishodišta, u i v rastu u suprotnom smjeru

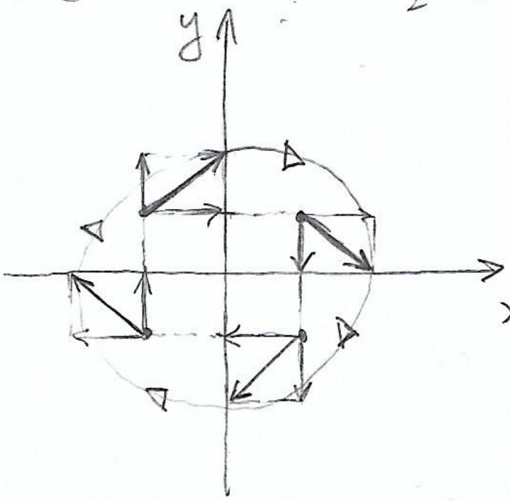
③ $\delta, D_1, D_2, u_0, v_0 = 0 \;;\; \zeta \neq 0$

3a) $\zeta > 0 \Rightarrow u = -\frac{1}{2}\zeta y \;;\; v = \frac{1}{2}\zeta x$



$\Rightarrow$ ANTICIKLONALNA VRTLOŽNOST

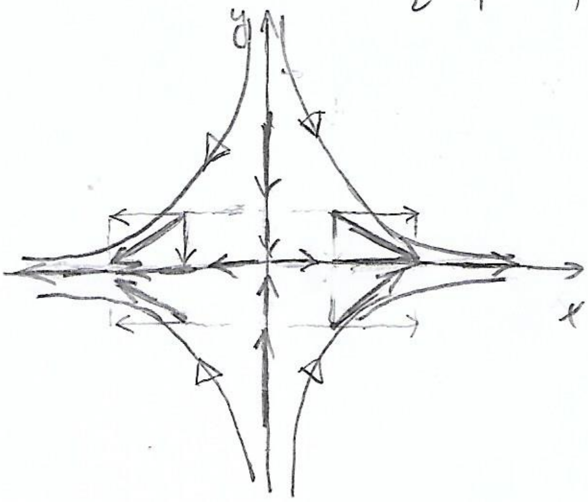
3b) $\xi < 0 \Rightarrow u = \frac{1}{2}|\xi| y \quad ; \quad v = -\frac{1}{2}|\xi| x$



$\Rightarrow$ CIKLONALNA VRTLOŽNOST

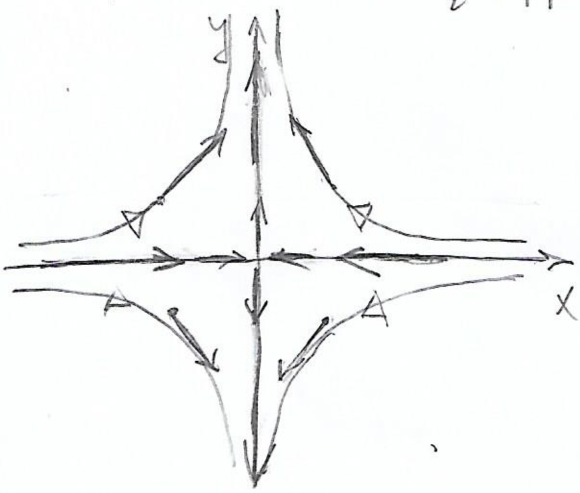
④ $\xi, \delta, D_2, u_0, v_0 = 0 \quad ; \quad D_1 \neq 0$

4a) $D_1 > 0 \Rightarrow u = \frac{1}{2} D_1 x \quad ; \quad v = -\frac{1}{2} D_1 y$



$\Rightarrow$ RASTEZANJE PO OSI x

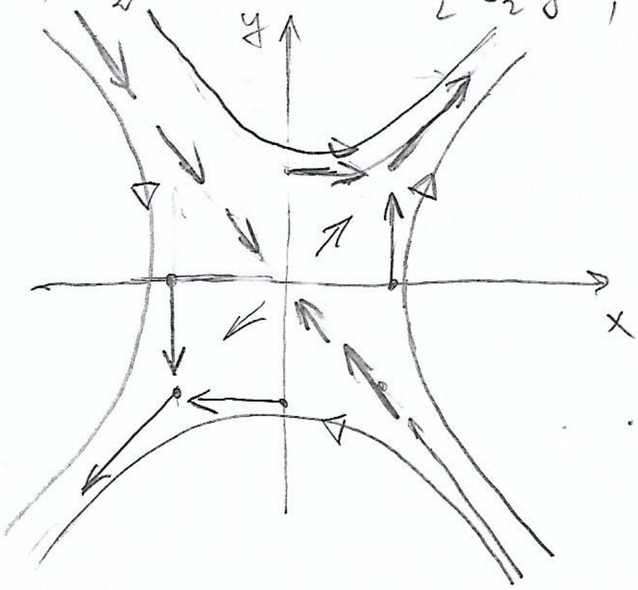
4b) $D_1 < 0 \Rightarrow u = -\frac{1}{2}|D_1| x \quad ; \quad v = \frac{1}{2}|D_1| y$



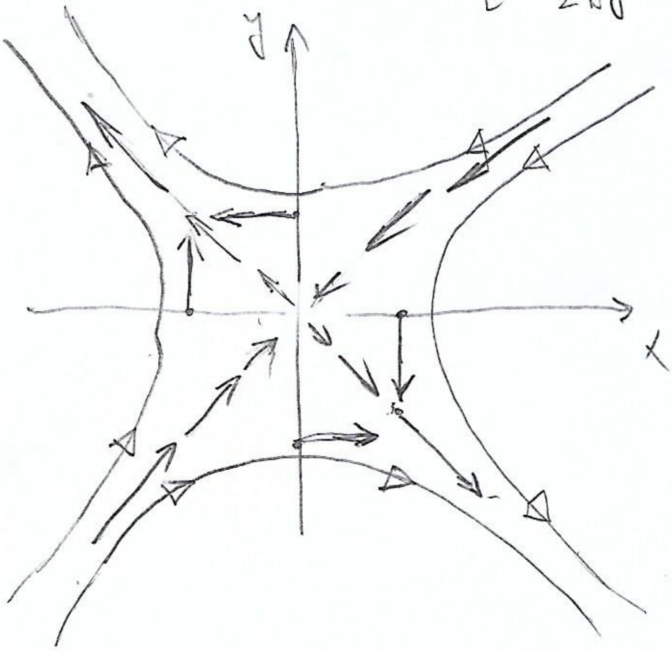
$\Rightarrow$ RASTEZANJE PO OSI y

⑤ $\delta, \xi, D_1, u_0, v_0 = 0 \;;\; D_2 \neq 0$

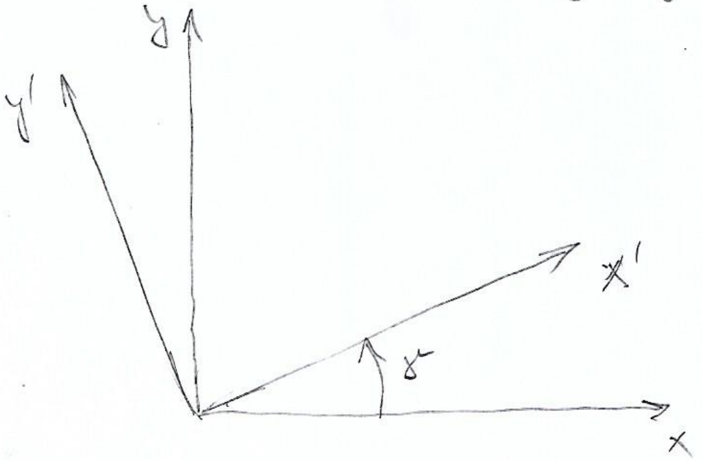
5a) $D_2 > 0 \Rightarrow u = \frac{1}{2} D_2 y \;;\; v = \frac{1}{2} D_2 x$



5b) $D_2 < 0 \Rightarrow u = -\frac{1}{2}|D_2| y \;;\; v = -\frac{1}{2}|D_2| x$



- u praksi katkad nije najpogodniji sustav xy već je pogodnije zarotirati sustav te se dobije x'y'



$$x' = x\cos\gamma + y\sin\gamma$$
$$y' = -x\sin\gamma + y\cos\gamma$$

$$x = x'\cos\gamma - y'\sin\gamma$$
$$y = x'\sin\gamma + y'\cos\gamma$$

- pokazuje se da u rotiranom sustavu vrijedi:

$\boxed{\delta' = \delta \; ; \; \xi' = \xi}$; $\boxed{\begin{aligned} D_1' &= D_1 \cos 2\gamma + D_2 \sin 2\gamma \\ D_2' &= -D_1 \sin 2\gamma + D_2 \cos 2\gamma \end{aligned}}$

- povoljno je sustav zarotirati t.d. je $D_2' = 0$

$$(1)\; D_1' = D_1 \cos 2\gamma_d + D_2 \sin 2\gamma_d$$

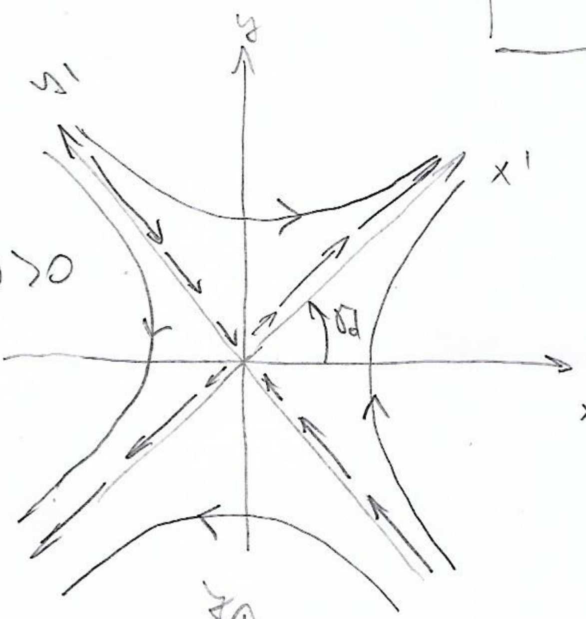
$$(2)\; 0 = -D_1 \sin 2\gamma_d + D_2 \cos 2\gamma_d$$

$$(2) \Rightarrow \boxed{\gamma_d = \frac{1}{2} \operatorname{arctg} \frac{D_2}{D_1}}$$

$$(1) / \frac{1}{\cos 2\gamma_d} \Rightarrow D_1' = \left(D_1 + \frac{D_2^2}{D_1}\right) \cos 2\gamma_d$$

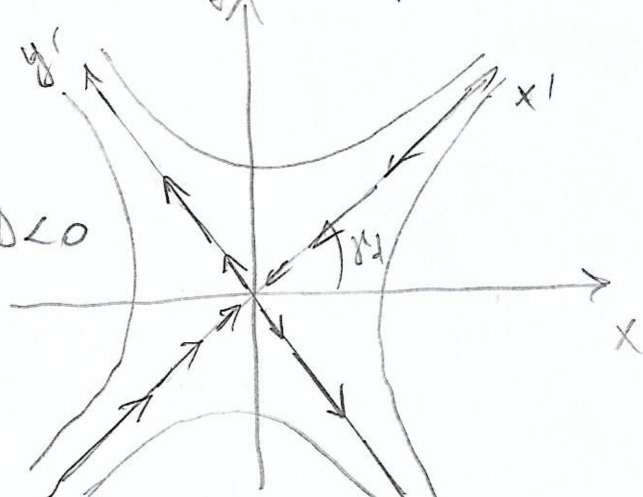
$$(2) \Rightarrow D_2 = D_1 \operatorname{tg} 2\gamma_d \Rightarrow \boxed{D_1' = \ldots = \frac{D_1}{\cos 2\gamma_d} = \frac{D_2}{\sin 2\gamma_d} = D}$$

D > 0



→ razvlačenje duž osi x' (os DILATACIJE) i sažimanje duž osi y' (os KONTRAKCIJE)

D < 0



→ ovdje je os y' os dilatacije, a os x' os kontrakcije

- polje vjetra u rotiranom sustavu za γ_d $(D_2' = 0)$:

$$u' = u_0' + \frac{1}{2}\delta' x' - \frac{1}{2}\zeta' y' + \frac{1}{2}D_1' x' + \frac{1}{2}D_2' y' = u_0' + \frac{1}{2}\delta x' - \frac{1}{2}\zeta y' + \frac{1}{2}D x'$$

$$v' = v_0' + \frac{1}{2}\delta' y' + \frac{1}{2}\zeta' x' - \frac{1}{2}D_1' y' + \frac{1}{2}D_2' x' = v_0' + \frac{1}{2}\delta y + \frac{1}{2}\zeta x' - \frac{1}{2}D y'$$

$$\frac{\partial u'}{\partial x'} = \frac{1}{2}(\delta + D) \qquad \frac{\partial v'}{\partial x'} = \frac{1}{2}\zeta$$

$$\frac{\partial u'}{\partial y'} = -\frac{1}{2}\zeta \qquad \frac{\partial v'}{\partial y'} = \frac{1}{2}(\delta - D)$$