IZVOD Q-VEKTORA ($\vec{Q}$) PREKO ω-JEDNADŽBE

- kvazigeostrofička ω-jednadžba:

$$\underbrace{\left(f_0^2 \frac{\partial^2}{\partial p^2} + \sigma \nabla^2\right)}_{\nabla^2}\omega = \frac{\partial}{\partial p} J\left(\phi, \frac{1}{f_0}\nabla^2\phi + f\right) + \nabla^2 J\left(\frac{1}{f_0}\phi, -\frac{\partial \phi}{\partial p}\right) \quad (*)$$

∇^2 ... "fizikalni laplasijan"

- promatrajmo ω na nekoj izobarnoj plohi i pridjelimo joj pravilnu sinusnu biperiodičku funkciju (za lakšu interpretaciju):

$$\omega = \omega_0 \sin(kx)\sin(ly)$$

$$\Rightarrow \nabla^2\omega \sim -(k^2+l^2)\omega \sim -\omega \sim w$$

- dakle, cijela desna strana ω-jednadžbe je proporcionalna s vertikalnom komponentom brzine w (u xyz sustavu)
- sada možemo pisati sljedeće:

$$w \sim J\left(\frac{\partial\phi}{\partial p}, \frac{1}{f_0}\nabla^2\phi + f\right) + J\left(\phi, \frac{1}{f_0}\frac{\partial}{\partial p}\nabla^2\phi\right) + J\left(\frac{1}{f_0}\nabla^2\phi, -\frac{\partial\phi}{\partial p}\right) + J\left(\frac{1}{f_0}\phi, -\nabla^2\frac{\partial\phi}{\partial p}\right)$$

RASPISANA DESNA STRANA U (*)

$$\Rightarrow w \sim J\left(\frac{\partial\phi}{\partial p}, \frac{1}{f_0}\nabla^2\phi + f\right) + \cancel{J\left(\phi, \frac{1}{f_0}\frac{\partial}{\partial p}\nabla^2\phi\right)} + J\left(\frac{\partial\phi}{\partial p}, \frac{1}{f_0}\nabla^2\phi\right) - \cancel{J\left(\frac{1}{f_0}\phi, \frac{\partial}{\partial p}\nabla^2\phi\right)}$$

$$\Rightarrow w \sim J\left(\frac{\partial\phi}{\partial p}, \frac{2}{f_0}\nabla^2\phi + f\right)$$

- kako vrijedi: $\frac{1}{f_0}\nabla^2\phi = \zeta_g$

$$\Rightarrow w \sim J\left(\frac{\partial\phi}{\partial p}, 2\zeta_g + f\right) = \vec{k}\cdot\left[\nabla\left(\frac{\partial\phi}{\partial p}\right) \times \nabla(2\zeta_g + f)\right] =$$

$$= \left(\vec{k} \times \nabla\frac{\partial\phi}{\partial p}\right)\cdot\nabla(2\zeta_g + f) = \frac{\partial}{\partial p}\left(\vec{k}\times\nabla\phi\right)\cdot\nabla(2\zeta_g+f) \ldots$$

- znamo: $\vec{V}_g = \frac{1}{f_0}\vec{k}\times\nabla\phi$

$$\dots\ w \sim f_0 \frac{\partial \vec{V}_g}{\partial p}\cdot\nabla(2\zeta_g + f) = \underbrace{2 f_0 \frac{\partial \vec{V}_g}{\partial p}\cdot\nabla\zeta_g}_{A} + \underbrace{f_0 \frac{\partial \vec{V}_g}{\partial p}\cdot\nabla f}_{B}$$

$$A\quad 2 f_0 \frac{\partial \vec{V}_g}{\partial p}\cdot\nabla\zeta_g = 2 f_0\left[\nabla\cdot\left(\zeta_g \frac{\partial \vec{V}_g}{\partial p}\right) - \zeta_g \nabla\cdot\frac{\partial \vec{V}_g}{\partial p}\right] =$$

$$= 2 f_0\left[\nabla\cdot\left(\zeta_g \frac{\partial \vec{V}_g}{\partial p}\right) - \zeta_g \frac{\partial}{\partial p}\cancelto{0}{\nabla\cdot\vec{V}_g}\right] =$$

$$= 2 f_0 \nabla\cdot\left(\zeta_g \frac{\partial \vec{V}_g}{\partial p}\right)$$

$$B\quad f_0 \frac{\partial \vec{V}_g}{\partial p}\cdot\nabla f = \cancel{f_0}\frac{\partial}{\partial p}\left(\frac{1}{\cancel{f_0}}\vec{k}\times\nabla\phi\right)\cdot\nabla f = \vec{k}\times\nabla\frac{\partial\phi}{\partial p}\cdot\nabla f$$

- kako vrijedi: $\frac{\partial\phi}{\partial p} = -\alpha = -\frac{RT}{p}$

$$\Rightarrow f_0 \frac{\partial \vec{V}_g}{\partial p}\cdot\nabla f = -\frac{R}{p}\vec{k}\times\nabla T\cdot\nabla f = -\frac{R}{p}\left[\vec{k}\times\left(\frac{\partial T}{\partial x}\vec{i} + \frac{\partial T}{\partial y}\vec{j}\right)\right]\cdot\nabla f =$$

$$= -\frac{R}{p}\left[\frac{\partial T}{\partial x}\vec{j} - \frac{\partial T}{\partial y}\vec{i}\right]\cdot\left(\cancelto{0}{\frac{\partial f}{\partial x}}\vec{i} + \underbrace{\frac{\partial f}{\partial y}}_{\beta}\vec{j}\right) = -\frac{R}{p}\beta\frac{\partial T}{\partial x}$$

$$\Rightarrow w \sim A + B = 2 f_0 \nabla\cdot\left(\zeta_g \frac{\partial \vec{V}_g}{\partial p}\right) - \frac{R}{p}\beta\frac{\partial T}{\partial x}$$

- dimenzionalna analiza pokazuje da je $B \ll A$ i to iz 2 razloga:
  1) $\beta \sim 10^{-12}$
  2) za planetarna gibanja se temperatura po paraleli (po x) značajno manje mijenja nego po meridijanu (po y)
- da budemo sigurni, koordinatni sustav postavimo tako da je os x paralelna izotermama $\Rightarrow \frac{\partial T}{\partial x} = 0 \Rightarrow B = 0$!
- sada uvodimo pokratu (Q-vektor): $\boxed{\vec{Q} = -f_0 \zeta_g \frac{\partial \vec{V}_g}{\partial p}}$

$$\Rightarrow \boxed{w \sim -2\nabla\cdot\vec{Q}}$$

DISKUSIJA $\vec{Q}$-a

- $\vec{Q}$ u sebi sadrži jednadžbu termalnog vjetra $\left(\frac{\partial \vec{V}_g}{\partial p} = -\frac{R}{p}\vec{k}\times\nabla T\right)$ te ukazuje na područja dizanja ili spuštanja zraka
- u praksi se $\vec{Q}$ izračunava (procjenjuje):

$$\nabla\cdot\vec{Q}\begin{cases} <0, & w>0 \Rightarrow \text{uzlazno gibanje} \\ >0, & w<0 \Rightarrow \text{silazno gibanje}\end{cases}$$

$$\nabla\cdot\vec{Q}\begin{cases} <0, & \text{konvergencija } \vec{Q}, \quad w>0 \\ >0, & \text{divergencija } \vec{Q}, \quad w<0\end{cases}$$

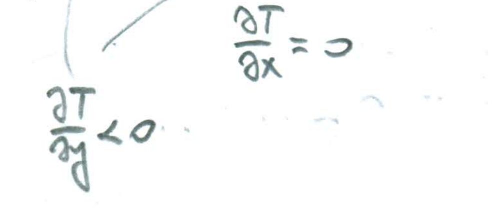
OPERATIVNA FORMULACIJA $\vec{Q}$-a

- Holton (2004) daje izvod $\vec{Q}$-vektora počevši od osnovnih geostrofičkih jednadžbi (str. 168-170)
- **DZ** Napraviti izvod $\vec{Q}$-vektora prema Holton (2004) uz uvažavanje adijabatske aproksimacije!
- Holton dolazi do sljedećeg izraza:

$$\vec{Q} = -\frac{R}{p}\left[\left(\frac{\partial \vec{V}_g}{\partial x}\cdot\nabla T\right)\vec{i} + \left(\frac{\partial \vec{V}_g}{\partial y}\cdot\nabla T\right)\vec{j}\right] = Q_1\vec{i} + Q_2\vec{j}$$

$$Q_1 = -\frac{R}{p}\frac{\partial \vec{V}_g}{\partial x}\cdot\nabla T \quad ; \quad Q_2 = -\frac{R}{p}\frac{\partial \vec{V}_g}{\partial y}\cdot\nabla T$$

- u prilagođenom koord. sustavu pogodnim za praćenje $\vec{Q}$-a, gdje je x os paralelna sa izotermama, a os y gleda u smjeru negativnog gradijenta temperature, vrijedi sljedeće:



$$\Rightarrow \nabla T = \underbrace{\frac{\partial T}{\partial x}}_{0}\vec{i} + \underbrace{\frac{\partial T}{\partial y}}_{<0}\vec{j}$$

$$\Rightarrow \nabla T = -\left|\frac{\partial T}{\partial y}\right|\vec{j}$$

$$\Rightarrow \vec{Q} = -\frac{R}{p}\frac{\partial \vec{V}_g}{\partial x}\cdot \nabla T\,\vec{i} - \frac{R}{p}\frac{\partial \vec{V}_g}{\partial y}\cdot \nabla T\,\vec{j} =$$

$$= -\frac{R}{p}\left(\underbrace{\frac{\partial u_g}{\partial x}\cancel{\frac{\partial T}{\partial x}}}_{0} + \frac{\partial v_g}{\partial x}\frac{\partial T}{\partial y}\right)\vec{i} - \frac{R}{p}\left(\underbrace{\frac{\partial u_g}{\partial y}\cancel{\frac{\partial T}{\partial x}}}_{0} + \frac{\partial v_g}{\partial y}\frac{\partial T}{\partial y}\right)\vec{j} =$$

$$= -\frac{R}{p}\frac{\partial v_g}{\partial x}\frac{\partial T}{\partial y}\vec{i} - \frac{R}{p}\frac{\partial v_g}{\partial y}\frac{\partial T}{\partial y}\vec{j} = -\frac{R}{p}\frac{\partial T}{\partial y}\left(\frac{\partial v_g}{\partial x}\vec{i} + \frac{\partial v_g}{\partial y}\vec{j}\right)$$

- jednadžba kontinuiteta za geostrofički vjetar:

$$\frac{\partial u_g}{\partial x} + \frac{\partial v_g}{\partial y} = 0 \Rightarrow \frac{\partial v_g}{\partial y} = -\frac{\partial u_g}{\partial x}$$

$$\Rightarrow \vec{Q} = -\frac{R}{p}\frac{\partial T}{\partial y}\underbrace{\left(\frac{\partial v_g}{\partial x}\vec{i} - \frac{\partial u_g}{\partial x}\vec{j}\right)}_{\frac{\partial \vec{V}_g}{\partial x}\times \vec{k}} = \frac{R}{p}\frac{\partial T}{\partial y}\vec{k}\times\frac{\partial \vec{V}_g}{\partial x}$$

$$\Rightarrow \vec{Q} = -\frac{R}{p}\left|\frac{\partial T}{\partial y}\right|\vec{k}\times\frac{\partial \vec{V}_g}{\partial x} \Rightarrow \boxed{\vec{Q} = \frac{R}{p}\left|\frac{\partial T}{\partial y}\right|(-\vec{k})\times\frac{\partial \vec{V}_g}{\partial x}}$$

operativna formula za procjenu $\vec{Q}$-a