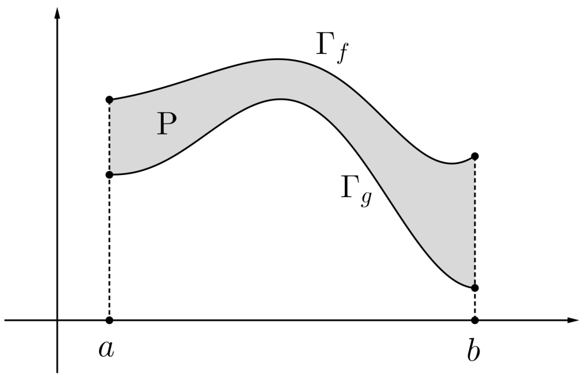
POVRŠINE LIKOVA

(a) Kartezijeve koordinate

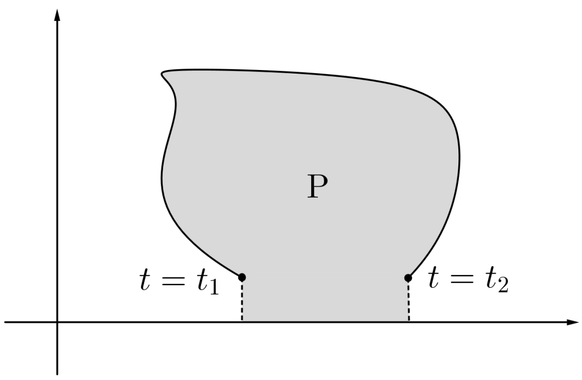
$$f, g : [a,b] \to \mathbb{R}, \quad f(x) \geq g(x); \quad x \in [a,b]$$



$$\mathrm{P} = \int_a^b (f(x) - g(x))\, dx$$

(b) Parametarski

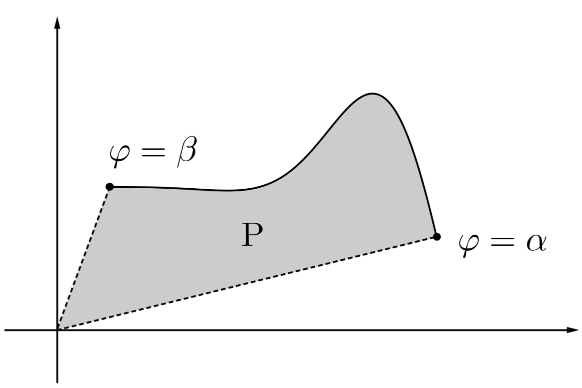
$$\begin{cases} x = x(t) \\ y = y(t) \end{cases} ; \quad t \in [t_1, t_2]$$



$$\mathrm{P} = \int_{t_1}^{t_2} y(t)\ \dot{x}(t)\ dt$$

(c) Polarne koordinate

$$r = f(\varphi)$$

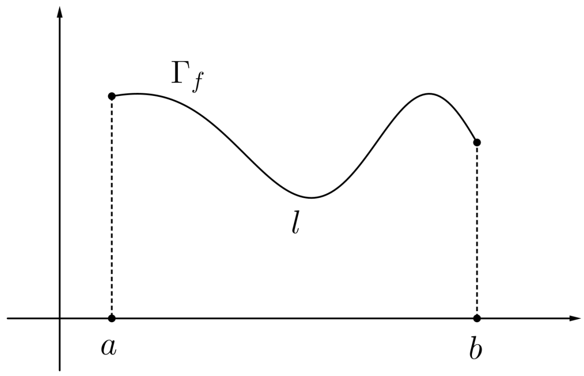


$$P = \frac{1}{2} \int_\alpha^\beta f(\varphi)^2\ d\varphi = \frac{1}{2} \int_\alpha^\beta r^2\ d\varphi$$

DULJINE LUKOVA KRIVULJA

(a) Kartezijeve koordinate

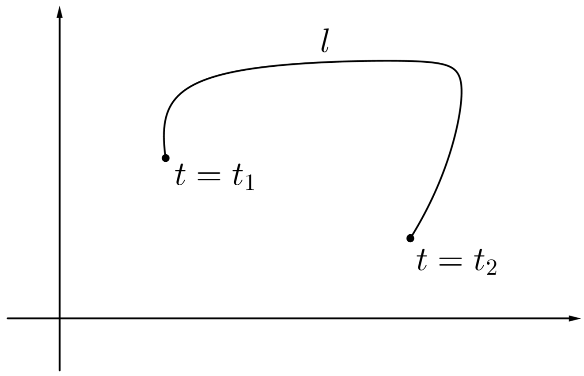
$$f : [a,b] \to \mathbb{R}$$



$$l = \int_a^b \sqrt{1 + f'(x)^2}\ dx$$

(b) Parametarski

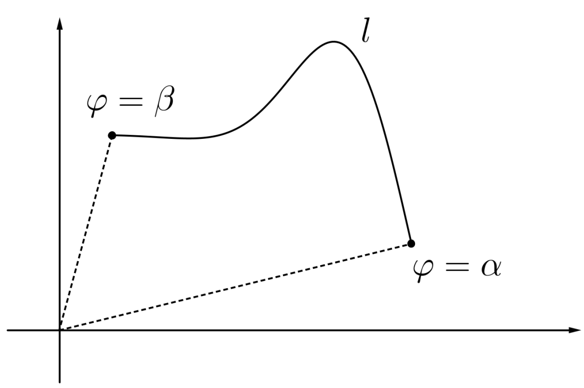
$$\begin{cases} x = x(t) \\ y = y(t) \end{cases} ; \quad t \in [t_1, t_2]$$



$$l = \int_{t_1}^{t_2} \sqrt{\dot{x}(t)^2 + \dot{y}(t)^2}\ dt$$

(c) Polarne koordinate

$$r = f(\varphi)$$



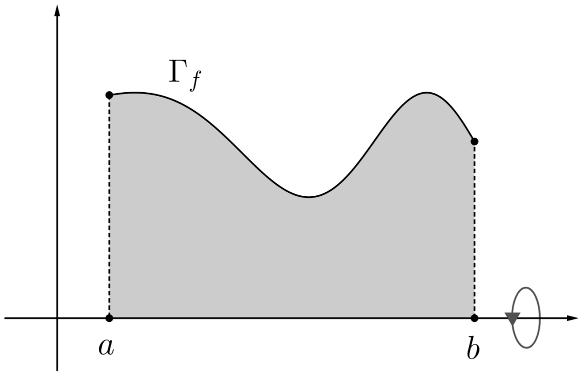
$$l = \int_\alpha^\beta \sqrt{f(\varphi)^2 + f'(\varphi)^2}\ d\varphi = \int_\alpha^\beta \sqrt{r^2 + \dot{r}^2}\ d\varphi$$

VOLUMENI ROTACIJSKIH TIJELA

(a) Kartezijeve koordinate

(1) Rotira oko x-osi

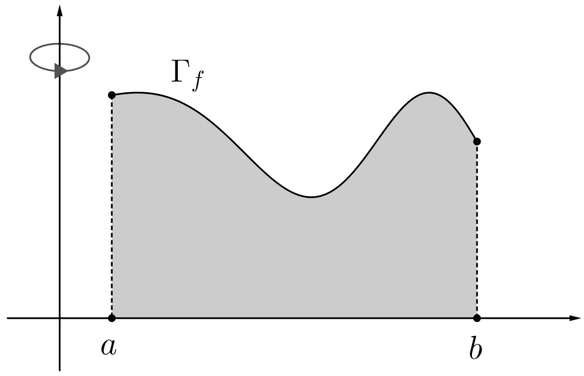
$$f : [a, b] \to \mathbb{R}$$



$$V_x = \pi \int_a^b f(x)^2 \, dx$$

(2) Rotira oko y-osi

$$0 \leq a < b, \quad f : [a, b] \to [0, +\infty\rangle$$

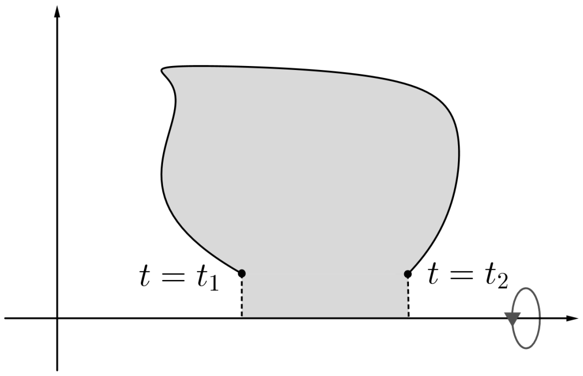


$$V_y = 2\pi \int_a^b x \, f(x) \, dx$$

(b) Parametarski

(1) Rotira oko x-osi

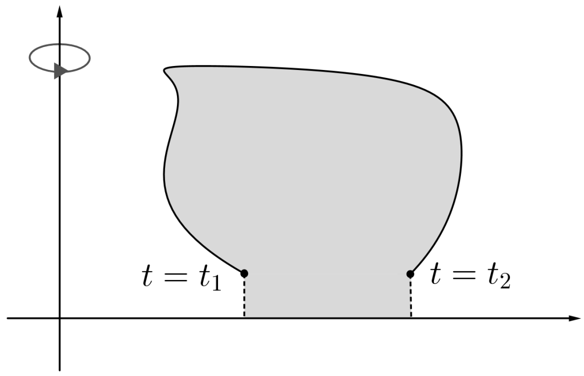
$$\begin{cases} x = x(t) \\ y = y(t) \end{cases} ; \quad t \in [t_1, t_2]$$



$$V_x = \pi \int_{t_1}^{t_2} y(t)^2 \, \dot{x}(t) \, dt$$

(2) Rotira oko y-osi

$$\begin{cases} x = x(t) \geq 0 \\ y = y(t) \geq 0 \end{cases} ; \quad t \in [t_1, t_2]$$

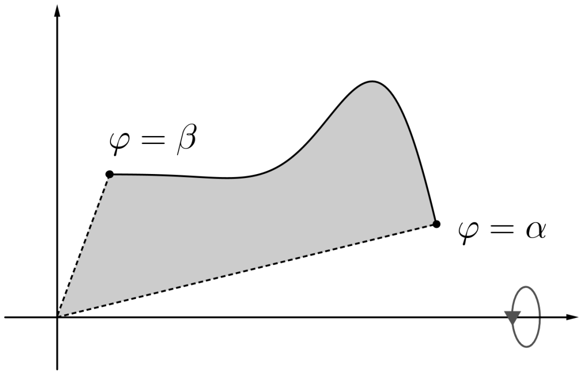


$$V_y = 2\pi \int_{t_1}^{t_2} x(t) \, \dot{x}(t) \, y(t) \, dt$$

(c) Polarne koordinate

(1) Rotira oko x-osi

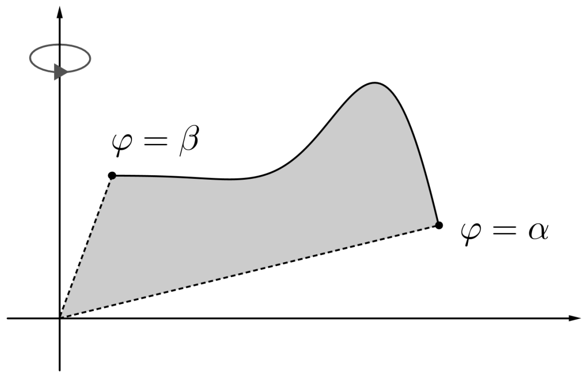
$$r = f(\varphi), \quad 0 \leq \alpha < \beta \leq \pi$$



$$V_x = \frac{2\pi}{3} \int_\alpha^\beta f(\varphi)^3 \, \sin\varphi \, d\varphi = \frac{2\pi}{3} \int_\alpha^\beta r^3 \, \sin\varphi \, d\varphi$$

(2) Rotira oko y-osi

$$r = f(\varphi), \quad -\frac{\pi}{2} \leq \alpha < \beta \leq \frac{\pi}{2}$$



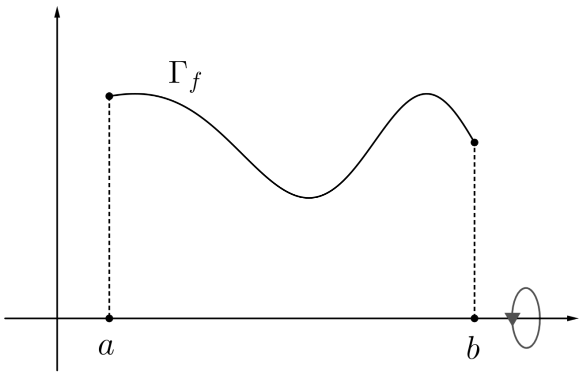
$$V_y = \frac{2\pi}{3} \int_\alpha^\beta f(\varphi)^3 \, \cos\varphi \, d\varphi = \frac{2\pi}{3} \int_\alpha^\beta r^3 \, \cos\varphi \, d\varphi$$

POVRŠINE ROTACIJSKIH PLOHA

(a) Kartezijeve koordinate

(1) Rotira oko x-osi

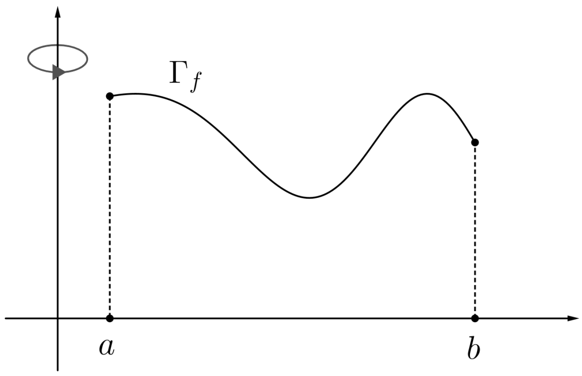
$$f : [a, b] \to [0, +\infty\rangle$$



$$S_x = 2\pi \int_a^b f(x)\sqrt{1 + f'(x)^2}\ dx$$

(2) Rotira oko y-osi

$$0 \leq a < b, \quad f : [a, b] \to \mathbb{R}$$

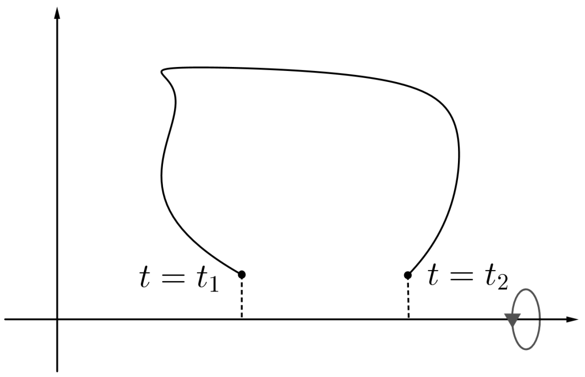


$$S_y = 2\pi \int_a^b x\sqrt{1 + f'(x)^2}\ dx$$

(b) Parametarski

(1) Rotira oko x-osi

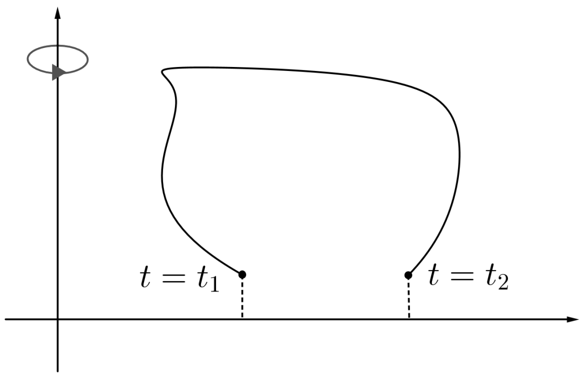
$$\begin{cases} x = x(t) \\ y = y(t) \geq 0 \end{cases} ; \quad t \in [t_1, t_2]$$



$$S_x = 2\pi \int_{t_1}^{t_2} y(t)\sqrt{\dot{x}(t)^2 + \dot{y}(t)^2}\ dt$$

(2) Rotira oko y-osi

$$\begin{cases} x = x(t) \geq 0 \\ y = y(t) \end{cases} ; \quad t \in [t_1, t_2]$$

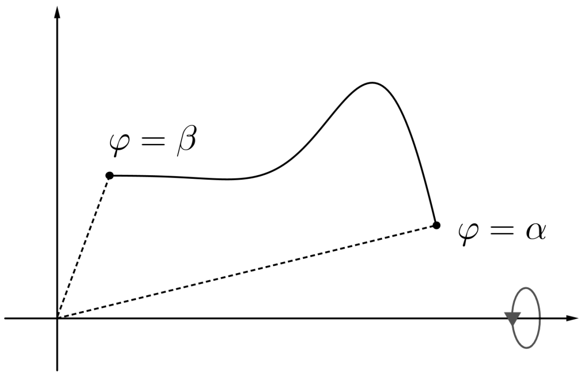


$$S_y = 2\pi \int_{t_1}^{t_2} x(t)\sqrt{\dot{x}(t)^2 + \dot{y}(t)^2}\ dt$$

(c) Polarne koordinate

(1) Rotira oko x-osi

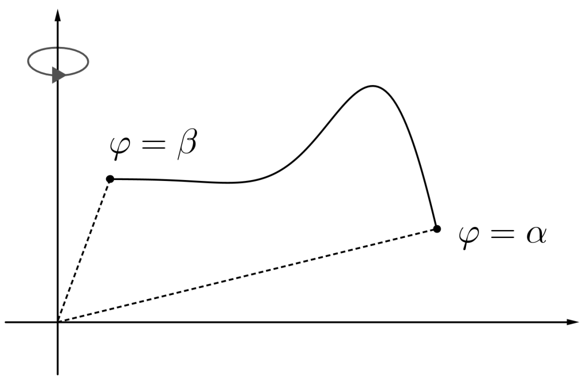
$$r = f(\varphi), \quad 0 \leq \alpha < \beta \leq \pi$$



$$S_x = 2\pi \int_\alpha^\beta f(\varphi)\ \sin\varphi\sqrt{f(\varphi)^2 + f'(\varphi)^2}\ d\varphi$$

(2) Rotira oko y-osi

$$r = f(\varphi), \quad -\frac{\pi}{2} \leq \alpha < \beta \leq \frac{\pi}{2}$$



$$S_y = 2\pi \int_\alpha^\beta f(\varphi)\ \cos\varphi\sqrt{f(\varphi)^2 + f'(\varphi)^2}\ d\varphi$$