PRIZEMNI SLOJ

- turbulentni tokovi topline i vlage očuvani (const)
- vrlo jake sile trenja i gradijenta tlaka
- $\frac{1}{20}$ visine PBL-a
- na prizemni sloj se primjenjuje teorija sličnosti
- vrijedi logaritamski zakon profila vjetra

[37] Koliko iznosi koeficijent turb. razmjene na visini 1 m ako je brzina vjetra na 2 m $2{,}6\,ms^{-1}$, a duljina hrapavosti $z_0 = 1$ cm. Van Karmanova konstanta je $k = 0{,}4$.

$\bar{u}(2m) = 2{,}6\,ms^{-1}$
$z_0 = 0{,}01\,m$
$k = 0{,}4$

$K = ?$

$$K = k \cdot z \cdot u_*$$

- logaritamski zakon je izveden od:

$$\frac{\partial \bar{u}}{\partial z} = \frac{u_*}{kz} \Big/ \int_{z_0}^{z} dz \Rightarrow \bar{u}(z) - \bar{u}(z_0) = \frac{u_*}{k} \ln \frac{z}{z_0}$$

($\bar{u}(z_0)$ = 0 po def.)

$$\Rightarrow \bar{u}(z) = \frac{u_*}{k} \ln \frac{z}{z_0} \Rightarrow u_* = k\,\bar{u}(z) \cdot \frac{1}{\ln \frac{z}{z_0}}$$

$$u_* = 0{,}4 \cdot 2{,}6\,ms^{-1} \cdot \frac{1}{\ln \frac{2m}{0{,}01m}} = 0{,}196\,ms^{-1}$$

$$K(z=1m) = 0{,}4 \cdot 1m \cdot 0{,}196\,ms^{-1} = 0{,}0785\,m^2 s^{-1}$$

[38] (*) Naći izraz za određivanje hrapavosti podloge z_0 pomoću brzine vjetra $\bar{u}_1$ i $\bar{u}_2$ na 2 visine z_1 i z_2 u neutralnom prizemnom sloju

$$\frac{\partial \bar{u}}{\partial z} = \frac{u^*}{kz} \Big/ \int_{z_0, z_0}^{z_1, z_2} dz \Rightarrow$$

$$(1)\ \bar{u}(z_1) = \frac{u^*}{k} \ln \frac{z_1}{z_0}$$

$$(2)\ \bar{u}(z_2) = \frac{u^*}{k} \ln \frac{z_2}{z_0}$$

$$\Rightarrow \frac{\bar{u}(z_1)}{\bar{u}(z_2)} = \frac{\ln\frac{z_1}{z_0}}{\ln\frac{z_2}{z_0}} \Rightarrow \frac{\bar{u}(z_1)}{\bar{u}(z_2)} \ln\frac{z_2}{z_0} = \ln\frac{z_1}{z_0}$$

$$\ln\left(\frac{z_2}{z_0}\right)^{\frac{\bar{u}(z_1)}{\bar{u}(z_2)}} = \ln\frac{z_1}{z_0} \quad /\exp \Rightarrow \left(\frac{z_2}{z_0}\right)^{\frac{\bar{u}(z_1)}{\bar{u}(z_2)}} = \frac{z_1}{z_0}$$

$$z_1 z_0^{\frac{\bar{u}(z_1)}{\bar{u}(z_2)}} = z_2^{\frac{\bar{u}(z_1)}{\bar{u}(z_2)}} z_0 \Rightarrow z_0^{\frac{\bar{u}(z_1)}{\bar{u}(z_2)}-1} = \frac{z_2^{\frac{\bar{u}(z_1)}{\bar{u}(z_2)}}}{z_1}$$

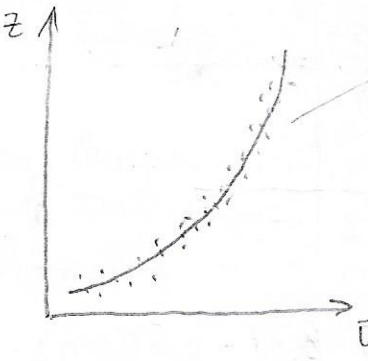
$$z_0^{\frac{\bar{u}(z_1)-\bar{u}(z_2)}{\bar{u}(z_2)}} = \frac{z_2^{\frac{\bar{u}(z_1)}{\bar{u}(z_2)}}}{z_1} \quad \Big/ \frac{\bar{u}(z_2)}{\bar{u}(z_1)-\bar{u}(z_2)}$$

$$\Rightarrow z_0 = \frac{z_2^{\frac{\bar{u}(z_1)}{\bar{u}(z_1)-\bar{u}(z_2)}}}{z_1^{\frac{\bar{u}(z_2)}{\bar{u}(z_1)-\bar{u}(z_2)}}} \quad //$$

39 Odredi brzinu trenja u_* i visinu hrapavosti z_0 na osnovu mnogo mjerenja $\bar{u}$ na različitim visinama z. Pp. da je sloj neutralan!

$$\boxed{\bar{u}(z) = \frac{u_*}{k}\ln\frac{z}{z_0}}$$

samo u neutralnom prizemnom sloju



tako to izgleda u prirodi

$$\bar{u}(z) = \frac{u_*}{k}\ln z - \frac{u_*}{k}\ln z_0$$

$$\bar{u}(z) = a\ln z - b$$

- koristimo metodu najmanjih kvadrata:

$$a = \frac{N\Sigma xy - \Sigma x\Sigma y}{N\Sigma x^2 - (\Sigma x)^2} \quad ; \quad b = \frac{\Sigma x^2\Sigma y - \Sigma x\Sigma xy}{N\Sigma x^2 - (\Sigma x)^2}$$

- kad izračunamo koeficijente a i b $\Rightarrow$

$$\Rightarrow a = \frac{u^*}{k} \Rightarrow \boxed{u^* = k \cdot a}$$

$$b = -\frac{u_*}{k} \ln z_0 = -a \ln z_0 \Rightarrow \ln z_0 = -\frac{b}{a} \Rightarrow \boxed{z_0 = e^{-\frac{b}{a}}}$$

(5.) Naći z_0 i koeficijent turb. razmjene K na visini 1 m te brzinu trenja u_* na temelju izmjerenih podataka u neutralnom prizemnom sloju

z [m]	$\bar{u}$ [m s^{-1}]	$\ln z$
0.5	1.4	-0,69
1.0	1.6	0
2.0	1.8	0,69
4.0	2.0	1,39
9.0	2.3	2,20
	y	x

$$\bar{u}(z) = \frac{u_*}{k} \ln \frac{z}{z_0}$$

$$\bar{u}(z) = \frac{u^*}{k} \ln z - \frac{u_*}{k} \ln z_0$$

$$y = ax + b$$

	x	y	xy	x^2
	-0,69	1,4	-0,97	0,48
	0	1,6	0	0
	0,69	1,8	1,24	0,48
	1,39	2,0	2,78	1,93
	2,20	2,3	5,06	4,84
Σ	3,59	9,1	8,11	7,73

$N = 5$

$$a = \frac{N\Sigma xy - \Sigma x \Sigma y}{N\Sigma x^2 - (\Sigma x)^2} = \frac{5 \cdot 8{,}11 - 3{,}59 \cdot 9{,}1}{5 \cdot 7{,}73 - 12{,}89} = 0{,}306\ s^{-1}$$

$$b = \frac{\Sigma x^2 \Sigma y - \Sigma x \Sigma xy}{N\Sigma x^2 - (\Sigma x)^2} = \frac{7{,}73 \cdot 9{,}1 - 3{,}59 \cdot 8{,}11}{5 \cdot 7{,}73 - 12{,}89} = 1{,}6005\ ms^{-1}$$

$$z_0 = e^{-\frac{b}{a}} = e^{-\frac{1{,}6005}{0{,}306}} = 0{,}0054\ m = 5{,}4\ mm$$

$$u_* = k \cdot a = 0{,}4 \cdot 0{,}306 = 0{,}12\ ms^{-1}$$

$$K = u_* k \cdot z \Rightarrow K(1m) = u_* \cdot k \cdot 1m = 0{,}048\ m^2 s^{-1}$$

- pogledajmo još horizontalni zakon

$\frac{\partial \bar{u}}{\partial z} = \frac{u_*}{kz}$ → može se bezdimenzionirati

$\Rightarrow \frac{\partial \bar{u}}{\partial z} \cdot \frac{kz}{u_*} = 1 = \phi_m$ → univerzalna fja sličnosti za količinu gibanja

- ako $\phi_m = 1 \Rightarrow$ neutralan prizemni granični sloj
- ako $\phi_m \neq 1 \Rightarrow$ ili stabilan ili nestabilan priz. gr. sloj
- stabilnost se gleda prema parametru $\frac{z}{L} = \zeta$

$$\phi_m\left(\frac{z}{L}\right) = \begin{cases} \left(1 - 15\frac{z}{L}\right)^{-\frac{1}{4}}, & \text{nestabilan PS}, \ \frac{z}{L} = \zeta < 0 \\ 1, & \text{neutralan PS}, \ \frac{z}{L} = \zeta = 0 \\ \left(1 + 4{,}7\frac{z}{L}\right), & \text{stabilni PS}, \ \frac{z}{L} = \zeta > 0 \end{cases}$$

- Richardsonov broj se koristi u Ekmanovom sloju, a parametar $\frac{z}{L}$ u prizemnom sloju (PS-u)
- isto vrijedi i za temperaturu i za vlagu

$$\frac{\partial \bar{T}}{\partial z} \cdot \frac{kz}{T_*} = \phi_h\left(\frac{z}{L}\right) \quad ; \quad \frac{\partial \bar{q}}{\partial z} \cdot \frac{kz}{q_*} = \phi_q\left(\frac{z}{L}\right) \approx \phi_h\left(\frac{z}{L}\right)$$

za temperaturu i vlagu se često uzimaju iste vrijednosti

$$\phi_h\left(\frac{z}{L}\right) = \begin{cases} 0{,}74\left(1 - 9\frac{z}{L}\right)^{-\frac{1}{2}}, & \text{nestabilan PS}, \ \zeta < 0 \\ 0{,}74, & \text{neutralan PS}, \ \zeta = 0 \\ 0{,}74 + 4{,}7\frac{z}{L}, & \text{stabilan PS}, \ \zeta > 0 \end{cases}$$

40. Na temelju izmjerenih vrijednosti pri tlu: $u_* = 0{,}2\,ms^{-1}$, $\frac{g}{\theta_v} = 0{,}0333\,ms^{-2}K^{-1}$, $\overline{w'\theta_v'} = -0{,}05\,Kms^{-1}$ i na 10 m visine: $\frac{\partial \bar{u}}{\partial z} = \frac{20\,ms^{-1}}{100\,m}$, $\frac{\partial \bar{\theta}_v}{\partial z} = \frac{20\,°C}{100\,m}$, nađite parametre L, ζ, ϕ_m, ϕ_h na visini $z = 10\,m$ i $f = 10^{-4}\,s^{-1}$ $(k = 0{,}4)$

$$L = -\frac{\overline{\theta_v}\, u_*^3}{k \cdot g \cdot (\overline{w'\theta_v'})}$$ → Monin-Obukovljeva duljina ⇒ pokazuje visinu na kojoj su dinamički i termički faktor generiranja turbulencije jednaki

Ako: $u_* = 0{,}2\, ms^{-1}$

$\frac{g}{\overline{\theta_v}} = 0{,}0333\, ms^{-2}K^{-1}$

$\overline{w'\theta_v'} = -0{,}05\, Kms^{-1}$

10 m: $\frac{\partial \overline{u}}{\partial z} = \frac{20\, ms^{-1}}{100\, m}$

$\frac{\partial \overline{\theta_v}}{\partial z} = \frac{20\,°C}{100\, m}$

$z = 10\, m$

$k = 0{,}4$

$$L = -\frac{\overline{\theta_v}}{g} \cdot \frac{u_*^3}{k\, \overline{w'\theta_v'}} =$$

$$= +\frac{1}{0{,}0333} \cdot \frac{0{,}2^3}{0{,}4 \cdot (+0{,}05)}$$

$$= 12\, m$$

$$\zeta(z = 10m) = \frac{z}{L} = \frac{10}{12} = 0{,}833 > 0$$ → stabilan PS

- budući da je $\zeta > 0$ → stabilno a to se vidi i po $\overline{w'\theta_v'} < 0$

- ako PS nije neutralan:

$$\phi_m\left(\frac{z}{L}\right) = \frac{\partial \overline{u}}{\partial z} \cdot \frac{kz}{u_*} \Rightarrow \phi_m\left(\frac{10}{L}\right) = \frac{20\, s^{-1}}{100} \cdot \frac{0{,}4 \cdot 10\, m}{0{,}2\, ms^{-1}} = 4$$

$$\phi_h\left(\frac{z}{L}\right) = \frac{\partial \overline{\theta_v}}{\partial z} \cdot \frac{kz}{\theta_*} = \left\{\theta_* = -\frac{\overline{w'\theta'}}{u_*} = \frac{0{,}05}{0{,}2} = 0{,}25\, K\right\} = 3{,}2$$

41. Pp. logaritamski profil vjetra. Poznato je $\overline{w'\theta'} = 0{,}2\, Kms^{-1}$, $z_i = 500\, m$, $\frac{g}{\overline{\theta}} = 0{,}0333\, ms^{-2}K^{-1}$, $z_0 = 0{,}01\, m$, $u_* = 0{,}2\, ms^{-1}$, $k = 0{,}4$, $z = 6\, m$, nema vlage ($\overline{\theta_v} = \overline{\theta}$)

Naći: a) L b) ζ c) w_* d) θ_* e) statičku stabilnost

f) R_f na 6 m (uzmi potrebne pp.)

g) R_i na 6 m h) dinamičku stabilnost

i) stanje toka

a) $$L = -\frac{\overline{\theta}}{g} \cdot \frac{u_*^3}{k\, \overline{w'\theta'}} = -\frac{1}{0{,}0333\, ms^{-2}K^{-1}} \cdot \frac{0{,}2^3\, m^3 s^{-3}}{0{,}4 \cdot 0{,}2\, Kms^{-1}} = -3\, m$$

b) $$\zeta = \frac{z}{L} = \frac{6m}{-3m} = -2$$

c) $w_* = \sqrt[3]{\frac{g \cdot z_i}{\theta_v}(\overline{w'\theta'})} = 1{,}49\,ms^{-1}$

d) $\theta_* = -\frac{\overline{w'\theta'}}{u_*} = -\frac{0{,}2\,Kms^{-1}}{0{,}2\,ms^{-1}} = -1\,K$

f) $R_f = \frac{\frac{g}{\theta_v} \cdot \overline{w'\theta_v'}}{\overline{u'w'}\,\frac{\partial \bar{u}}{\partial z}} = \frac{\frac{g}{\theta_v}\,\overline{w'\theta'}}{-u_*^2 \cdot \frac{u_*}{kz}} = -1{,}998$

g) $R_i = \frac{\frac{g}{\theta_v}\,\frac{\partial \bar{\theta}_v}{\partial z}}{\left(\frac{\partial \bar{u}}{\partial z}\right)^2} = \frac{\frac{g}{\theta} \cdot \frac{\theta_*}{kz}}{\left(\frac{u_*}{kz}\right)^2} = \frac{\frac{g}{\theta} \cdot \left(-\frac{\overline{w'\theta'}}{u_* kz}\right)}{\frac{u_*^2}{(kz)^2}} = -\frac{g}{\theta} \cdot \frac{kz\,\overline{w'\theta'}}{u_*^2} = R_f$

h) $R_f < 0 \Rightarrow$ statički i dinamički nestabilno

i) turbulentan tok!