zad 4, poglavlje 3.3.

$$P_1(t) = 2t^2 - 1, \quad P_2(t) = t + t^2,$$

$$P_3(t) = 1 + 3t + 2t^2$$

$$\alpha P_1(t) + \beta P_2(t) + \gamma P_2(t) = 0$$

$$\alpha (2t^2 - 1) + \beta (t + t^2) + \gamma (1 + 3t + 2t^2) = 0$$

$$(2\alpha + \beta + 2\gamma) t^2 + (\beta + 3\gamma) t + (\gamma - \alpha) = 0$$

$$\begin{cases} 2\alpha + \beta + 2\gamma = 0 \\ \beta + 3\gamma = 0 \Rightarrow \beta = -3\gamma \\ \gamma - \alpha = 0 \Rightarrow \alpha = \gamma \end{cases}$$

$$\Rightarrow 2\gamma - 3\gamma + 2\gamma = 0 \Rightarrow \gamma = 0$$

$$\Rightarrow \alpha = \beta = 0$$

$\Rightarrow \{P_1, P_2, P_3\}$ je lin. nezavisan skup

$$ct^2+bt+a = \alpha' p_1(t) + \beta' p_2(t) + \gamma' p_3(t)$$

$$= \alpha'(2t^2-1) + \beta'(t+t^2) + \gamma'(1+3t+2t^2)$$

$$= (2\alpha'+\beta'+2\gamma')t^2 + (\beta'+3\gamma')t + (-\alpha'+\gamma')$$

$$\Longrightarrow \begin{aligned} 2\alpha'+\beta'+2\gamma' &= c \\ \beta'+3\gamma' &= b \quad \Rightarrow \beta' = b-3\gamma' \\ -\alpha'+\gamma' &= a \quad \Rightarrow \alpha' = \gamma'-a \end{aligned}$$

$$2(\gamma'-a)+b-3\gamma'+2\gamma' = c$$

$$\Rightarrow \gamma'-2a+b = c \Rightarrow \boxed{\gamma' = c-b+2a}$$

$$\boxed{\alpha' = a-b+c}$$

$$\boxed{\beta' = -3c+4b-6a}$$

$$ct^2+bt+a = (a-b+c)\,p_1(t) + (-3c+4b-6a)\,p_2(t) + (c-b+2a)\,p_3(t)$$

$$p_1^*(ct^2+bt+a) = a-b+c$$

$$p_2^*(ct^2+bt+a) = -3c+4b-6a$$

$$p_3^*(ct^2+bt+a) = c-b+2a$$

2. način (koristeći matrica)

$$p_1^* : \mathcal{P}_2(\mathbb{R}) \to \mathbb{R}$$

$(f) = \{1\}$ baza za $\mathbb{R}$

$(p) = \{p_1, p_2, p_3\}$

$(e) = \{1, t, t^2\}$

$$[p_1^*]_{(f,p)} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \qquad [p_3^*]_{(f,p)} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

$$[p_2^*]_{(f,p)} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

Zanima nas $[P_i^*]_{(f,e)}$ za $i=\{1,2,3\}$

$$[P_i^*]_{(f,e)} = [P_i^*]_{(f,P)} \cdot [I]_{(P,e)}$$

$$[P_1^*]_{(f,e)} = [1 \ 0 \ 0] \cdot [I]_{(e,P)}^{-1}$$

$$[P_2^*]_{(f,e)} = [0 \ 1 \ 0] \cdot [I]_{(e,P)}^{-1}$$

$$[P_3^*]_{(f,e)} = [0 \ 0 \ 1] \cdot [I]_{(e,P)}^{-1}$$

$$[I]_{(e,P)} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 3 \\ 2 & 1 & 2 \end{bmatrix}$$

$$[I]_{(P,e)} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 3 \\ 2 & 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ -6 & 4 & -3 \\ 2 & -1 & 1 \end{bmatrix}$$

$$\Rightarrow [P_1^*]_{(f,e)} = [1 \ -1 \ 1]$$

$$\Rightarrow P_1^*(a+bt+ct^2) = a-b+c$$

$$[P_2^*]_{(f,e)} = [-6 \quad 4 \quad -3]$$

$$\Rightarrow P_2^*(a+bt+ct^2) = -6a+4b-3c$$

$$[P_3^*]_{(f,e)} = [2 \quad -1 \quad 1]$$

$$\Rightarrow P_3^*(a+bt+ct^2) = 2a-b+c$$

$$f = \alpha P_1^* + \beta P_2^* + \gamma P_3^*$$

$$f(P_1) = (\alpha P_1^* + \beta P_2^* + \gamma P_3^*)(P_1) = \alpha$$

$$f(P_2) = \beta$$

$$f(P_3) = \gamma$$

$$\left.\begin{aligned} \alpha &= f(P_1) = \int_{-1}^{1} P_1(2t-1)\,dt \\ \beta &= f(P_2) = \int_{-1} P_2(2t-1)\,dt \\ \gamma &= f(P_3) = \int_{-1} P_3(2t-1)\,dt \end{aligned}\right\} \ldots \text{uvrstite i izračunajt}$$