

zad 5, cjelina 3.3. U prostoru  $(\mathbb{R}^3)^*$

dana je baza  $(a^*) = \{a_1^*, a_2^*, a_3^*\}$ ,

pri čemu je

$$a_1^*(x, y, z) = 2x,$$

$$a_2^*(x, y, z) = x + y + z$$

$$a_3^*(x, y, z) = x - 2y - 5z$$

Ođvedite bazu  $(a) = \{a_1, a_2, a_3\}$  za  $\mathbb{R}^3$   
kojoj je  $(a^*)$  dualna baza.

ij:

$$a_1 = (A, B, C), \quad a_2 = (D, E, F), \quad a_3 = (G, H, I)$$

Tržiimo  $a_1, a_2, a_3$ .

Vrijedi:

$$a_1^*(a_1) = 1 \Rightarrow 2A = 1$$

$$a_2^*(a_1) = 0 \Rightarrow A + B + C = 0$$

$$a_3^*(a_1) = 0 \Rightarrow A - 2B - 5C = 0$$

$$A = \frac{1}{2}$$

$$B = -1$$

$$C = \frac{1}{2}$$

$$a_1^* (a_2) = 0 \Rightarrow 2D = 0$$

$$a_2^* (a_2) = 1 \Rightarrow D + E + F = 1$$

$$a_3^* (a_2) = 0 \Rightarrow D - 2E - 5F = 0$$

$$D = 0$$

$$E = \frac{5}{3}$$

$$F = -\frac{2}{3}$$

$$a_1^* (a_3) = 0 \Rightarrow 2G = 0$$

$$a_2^* (a_3) = 0 \Rightarrow G + H + I = 0$$

$$a_3^* (a_3) = 1 \Rightarrow G - 2H - 5I = 1$$

$$G = 0$$

$$H = \frac{1}{3}$$

$$I = -\frac{1}{3}$$

$$a_1 = \left( \frac{1}{2}, -1, \frac{1}{2} \right)$$

$$\Rightarrow a_2 = \left( 0, \frac{5}{3}, -\frac{2}{3} \right)$$

$$a_3 = \left( 0, \frac{1}{3}, -\frac{1}{3} \right)$$

## 2. način (konistići matrice)

$$a_1^* : \mathbb{R}^3 \rightarrow \mathbb{R} \rightarrow (f) = \{1\} \text{ baza za } \mathbb{R}$$

$$[a_1^*]_{(f, a)} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

$\uparrow \quad \uparrow \quad \uparrow$   
 $a_1^*(a_1) \quad a_1^*(a_2) \quad a_1^*(a_3)$

$$[a_1^*]_{(f, e)} = \begin{bmatrix} 2 & 0 & 0 \end{bmatrix}$$

$\uparrow \quad \uparrow \quad \uparrow$   
 $a_1^*(e_1) \quad a_1^*(e_2) \quad a_1^*(e_3)$

$$(e) = \left\{ \underbrace{(1, 0, 0)}_{e_1}, \underbrace{(0, 1, 0)}_{e_2}, \underbrace{(0, 0, 1)}_{e_3} \right\}$$

$$[a_1^*]_{(f, a)} = [a_1^*]_{(f, e)} \cdot [I]_{(e, a)}$$

Analogno  $\underline{= [0 \ 1 \ 0]} \quad \underline{= [1 \ 1 \ 1]}$

$$[a_2^*]_{(f, a)} = [a_2^*]_{(f, e)} \cdot [I]_{(e, a)}$$

$$[a_3^*]_{(f, a)} = [a_3^*]_{(f, e)} \cdot [I]_{(e, a)}$$

$$\Rightarrow \begin{cases} [1 \ 0 \ 0] = [2 \ 0 \ 0] [I]_{(e, a)} \\ [0 \ 1 \ 0] = [1 \ 1 \ 1] [I]_{(e, a)} \\ [0 \ 0 \ 1] = [1 \ -2 \ -5] [I]_{(e, a)} \end{cases}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & -2 & -5 \end{bmatrix} \cdot [I]_{(e, a)}$$

$$\Rightarrow [I]_{(e, a)}^{-1} = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & -2 & -5 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -1 & \frac{5}{3} & \frac{1}{3} \\ \frac{1}{2} & -\frac{2}{3} & -\frac{1}{3} \end{bmatrix}$$

$a_1$        $a_2$        $a_3$

$$a_1 = \left( \frac{1}{2}, -1, \frac{1}{2} \right)$$

$$a_2 = \left( 0, \frac{5}{3}, -\frac{2}{3} \right)$$

$$a_3 = \left( 0, \frac{1}{3}, -\frac{1}{3} \right)$$