2018/2019

stupanj strogo manji od n

$A: \mathcal{P}_n \to M_2(\mathbb{R})$ $\dim \mathcal{P}_n = n$

$$A(p) = \begin{bmatrix} p'''(1) & p(1) \\ p'(1) & \frac{1}{2}p''(1) - p(0) \end{bmatrix}$$

Odredite sve $n \in \mathbb{N}$ za koje je $d(A) = 0$.

Rj: $\operatorname{Ker} A = \{ p \in \mathcal{P}_n \mid A(p) = 0_{M_2(\mathbb{R})} \}$

$p \in \operatorname{Ker} A \iff$
$$p'''(1) = 0$$
$$p(1) = 0$$
$$p'(1) = 0$$
$$\frac{1}{2}p''(1) - p(0) = 0$$

tm. o rangu i defektu

$d(A) = 0 \iff r(A) = n$

Uočimo da je $\operatorname{Im}(A) \leq M_2(\mathbb{R})$ pa je $r(A) \leq 4$.

Dakle, za $n \geq 5$ je sigurno $d(A) \neq 0$.

- Za $n = 4$: $p(t) = a_3t^3 + a_2t^2 + a_1t + a_0$

$p \in \operatorname{Ker} A \iff$
$$6a_3 = 0$$
$$a_3 + a_2 + a_1 + a_0 = 0$$
$$3a_3 + 2a_2 + a_1 = 0$$
$$\frac{1}{2}(6a_3 + 2a_2) - a_0 = 0$$

$\iff$
$$a_3 = 0$$
$$a_2 + a_1 + a_0 = 0$$
$$2a_2 + a_1 = 0$$
$$a_2 - a_0 = 0$$

$p \in \ker A \iff a_3 = 0,\ 2a_0 + a_1 = 0$ i $a_2 = a_0$

$\iff a_3 = 0,\ a_2 = a_0$ & $a_1 = -2a_0$

$\ker A = \{ a_0 (t^2 - 2t + 1) \mid a_0 \in \mathbb{R} \}$

$d(A) = 1$

- Za $n = 3$ (vidjeti a) dio zadatka)

- Za $n = 2$ $\quad p(t) = a_1 t + a_0$

$p \in \ker A \iff$
$$a_1 + a_0 = 0$$
$$a_1 = 0$$
$$-a_0 = 0$$

$\ker A = \{0\}$ $\quad d(A) = 0$

- Za $n = 1$ $\quad p(t) = a_0$

$p \in \ker A \iff$
$$a_0 = 0$$
$$-a_0 = 0$$

$\ker A = \{0\}$ $\quad d(A) = 0$

Dakle, $d(A) = 0$ za $n = 1$, $n = 2$ (i eventualno $n = 3$ (riješite a) dio zadatka sami :)).