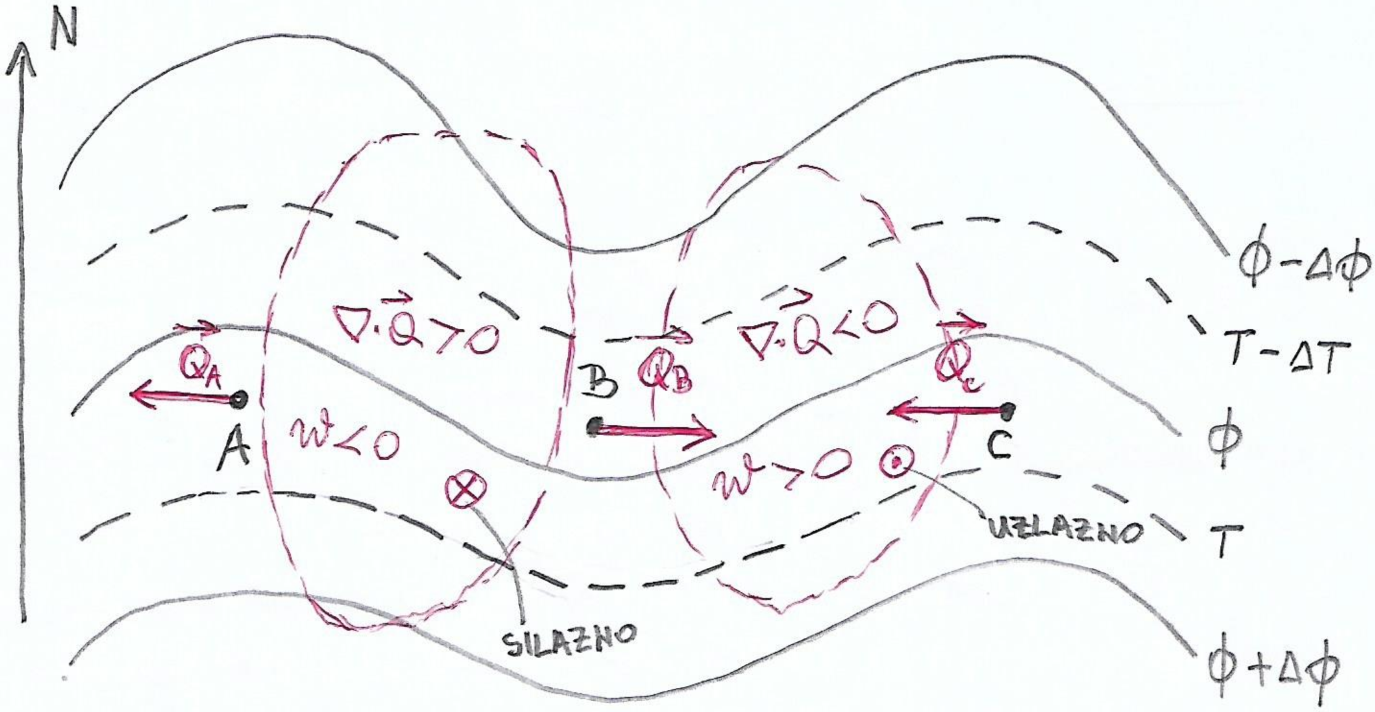
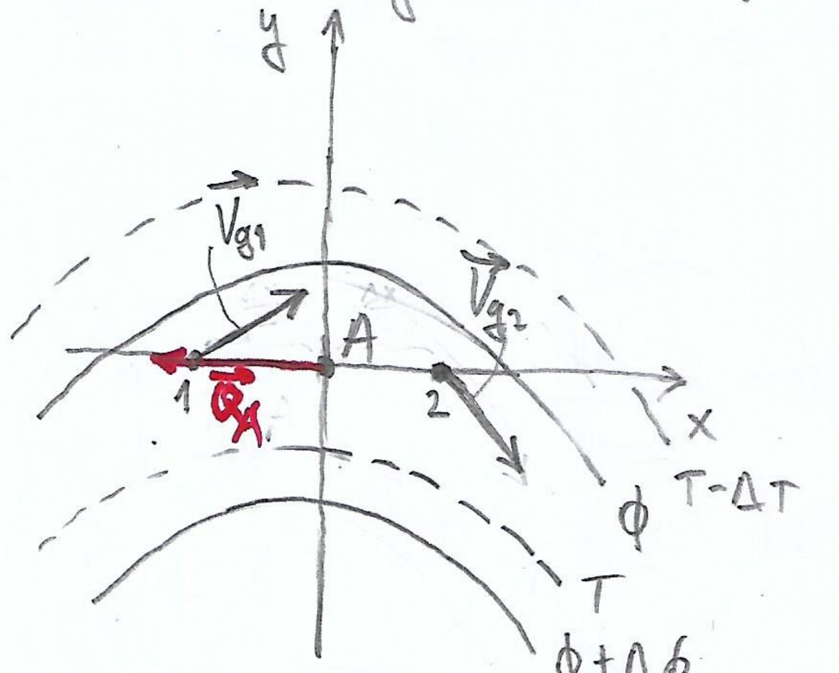
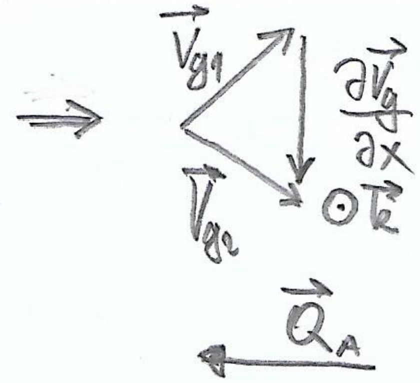
3. Procijenite polje $\vec{Q}$-vektora na slici ispod (u točkama A, B i C). Odredite područja uzdizanja i spuštanja zraka.



Rj: Točka A $\Rightarrow$ koordinatni sustav sa y-osi u smjeru opadanja temperature (u našem slučaju prema sjeveru, a x-os desno od y osi. Odaberemo 2 točke na x-osi, lijevo i desno od točke A kako bismo mogli procijeniti $\frac{\partial \vec{V}_g}{\partial x}$. Budući da koristimo konačne razlike, udaljenost između točaka 1 i 2 označimo sa Δx, pa je tada udaljenost između pojedine točke i točke A $\frac{\Delta x}{2}$. U točke 1 i 2 ucrtajmo vektore geostrofičkog vjetra $\Rightarrow$ paralelno s izolinijama geopotencijala



na način da je veći geopotencijal s desne strane. Vektori $\vec{V}_{g1}$ i $\vec{V}_{g2}$ su iste duljine, ali različitog smjera. Za procjenu $\frac{\partial \vec{V}_g}{\partial x}$ se opet pitamo: koji vektor moramo dodati vektoru $\vec{V}_{g1}$ da bismo došli do vektora $\vec{V}_{g2}$?

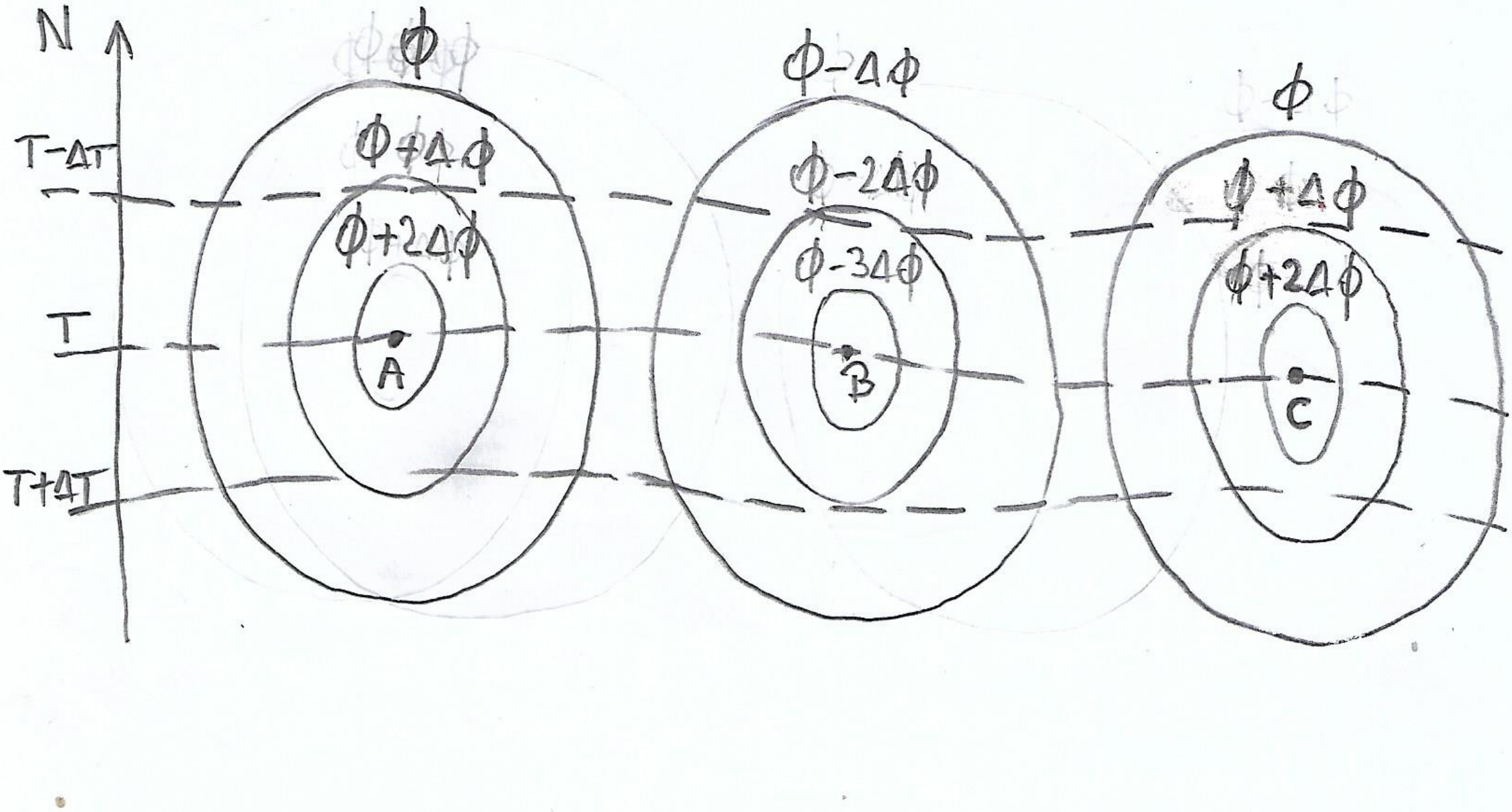


$\Rightarrow$ dakle, vektor $\frac{\partial \vec{V}_g}{\partial x}$ je orijentiran prema jugu u točki A na skici

- stoga je vektorski produkt $(-\vec{k})\times\frac{\partial \vec{V}_g}{\partial x}$ zbog predznaka (-) orijentiran prema zapadu ! (u neg. smjeru x osi)

- na isti način napravite procjenu $\vec{Q}$-vektora u točkama B i C te na kraju ucrtajte dobivene vektore na početnu skicu (ucrtani crvenom bojom). Vidimo da u području između točaka A i B $\vec{Q}$ vektor divergira ($\nabla\cdot\vec{Q} > 0 \Rightarrow w < 0$) pa je to područje spuštanja zraka, a između točaka B i C $\vec{Q}$-vektor konvergira ($\nabla\cdot\vec{Q} < 0 \Rightarrow w > 0$) pa je to područje uzdizanja zraka. To naznačimo na početnoj skici i zadatak je riješen!

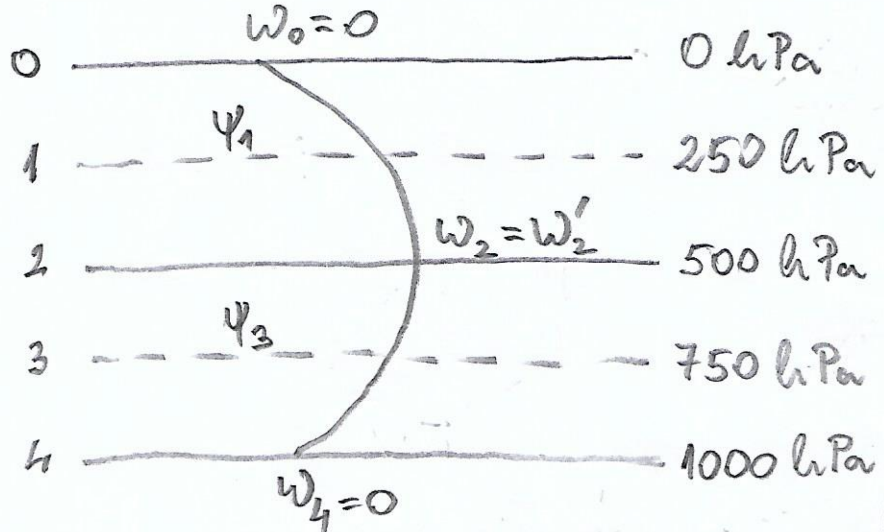
$\boxed{4}$ $\boxed{DZ}$ Procijenite polje $\vec{Q}$-vektora na skici ispod (u točkama A, B i C) te odredite područja uzdizanja i spuštanja zraka.



5 DVOSLOJNI MODEL I $\vec{Q}$-VEKTOR: Pokažite da u dvoslojnom modelu $\vec{Q}$-vektor i ω-jednadžba poprimaju sljedeće oblike:

$$\vec{Q} = \frac{2f_0}{\delta p} U_T \xi_2' \vec{i} \quad ; \quad \left(\frac{\partial^2}{\partial x^2} - 2\lambda^2\right)\omega_2' = -4\frac{f_0 U_T}{\delta p}\frac{\partial \xi_2}{\partial x}$$

Rj:



- Vrijedi: $\omega_2 = \cancelto{0}{\bar{\omega}_2} + \omega_2' = \omega_2'$

$\psi_1(x,y,t) = \bar{\psi}_1(y) + \psi_1'(x,t)$

$\psi_3(x,y,t) = \bar{\psi}_3(y) + \psi_3'(x,t)$

$$\psi = \frac{1}{f_0}\phi$$

(*)
$$\psi_1 = -U_1 y + \psi_1'(x,t)$$
$$\psi_3 = -U_3 y + \psi_3'(x,t)$$

- ω-jdžba u općenitom obliku preko $\vec{Q}$-vektora:

$$\left(\sigma\nabla_p^2 + f_0^2\frac{\partial^2}{\partial p^2}\right)\omega = -2\nabla_p\cdot\vec{Q}$$

- raspisujemo član $\frac{\partial^2 \omega}{\partial p^2} = \frac{\left(\frac{\partial\omega}{\partial p}\right)_3 - \left(\frac{\partial\omega}{\partial p}\right)_1}{\delta p} = \frac{\frac{\cancelto{0}{\omega_4} - \omega_2}{\delta p} - \frac{\omega_2 - \cancelto{0}{\omega_0}}{\delta p}}{\delta p} = -\frac{2\omega_2'}{(\delta p)^2}$

$\Rightarrow \left\{\sigma\nabla_p^2 + f_0^2\left[-\frac{2}{(\delta p)^2}\right]\right\}\omega_2' = -2\nabla_p\cdot\vec{Q}$; - u xz ravnini $\nabla_p^2 = \frac{\partial^2}{\partial x^2}$

$\Rightarrow \sigma\left[\frac{\partial^2}{\partial x^2} - 2\frac{f_0^2}{\sigma(\delta p)^2}\right]\omega_2' = -2\nabla_p\cdot\vec{Q}$ (**) ⇝ to sada ostavimo za kasnije, a nađimo $\vec{Q}$-vektor:

- Holton uvodi sljedeći oblik za $\vec{Q}$-vektor:

$$\vec{Q} = -\frac{R}{p}\frac{\partial\vec{V}_g}{\partial x}\cdot\nabla T\,\vec{i} - \frac{R}{p}\frac{\partial\vec{V}_g}{\partial y}\cdot\nabla T\,\vec{j} \Rightarrow \vec{Q} = \frac{\partial\vec{V}_g}{\partial x}\cdot\nabla\left(-\frac{R}{p}T\right)\vec{i} + \frac{\partial\vec{V}_g}{\partial y}\left(-\frac{R}{p}T\right)\vec{j}$$

$$-\frac{R}{p}T = -\alpha = \frac{\partial\Phi}{\partial p} \Rightarrow \vec{Q} = \frac{\partial\vec{V}_g}{\partial x}\cdot\nabla\left(\frac{\partial\Phi}{\partial p}\right)\vec{i} + \frac{\partial\vec{V}_g}{\partial y}\cdot\nabla\left(\frac{\partial\Phi}{\partial p}\right)\vec{j}$$

- na plohi 2 vrijedi: $\left(\frac{\partial\Phi}{\partial p}\right)_2 \approx \frac{\Phi_3 - \Phi_1}{\delta p} = f_0\frac{\psi_3 - \psi_1}{\delta p}$

$$\Rightarrow \vec{Q} = \frac{f_0}{\delta p}\left[-\frac{\partial\vec{V}_g}{\partial x}\cdot\nabla(\psi_1 - \psi_3)\vec{i} - \frac{\partial\vec{V}_g}{\partial y}\cdot\nabla(\psi_1 - \psi_3)\vec{j}\right]$$

- znamo da u dvoslojnom modelu vrijedi sljedeće:

$$\vec{V}_g = \vec{k} \times \nabla\psi \quad \text{i} \quad \vec{V}_g = \vec{V}_2 \Rightarrow \vec{V}_2 = \vec{k} \times \nabla\psi_2 \; ; \; \psi_2 = \frac{\psi_1 + \psi_3}{2}$$

$$\Rightarrow \vec{V}_2 = \frac{1}{2}\vec{k} \times \nabla(\psi_1 + \psi_3)$$

$$\Rightarrow \vec{V}_2 = \frac{1}{2}\left(\vec{k}\times\nabla\psi_1 + \vec{k}\times\nabla\psi_3\right) = \frac{1}{2}\left(\frac{\partial\psi_1}{\partial x}\vec{j} - \frac{\partial\psi_1}{\partial y}\vec{i} + \frac{\partial\psi_3}{\partial x}\vec{j} - \frac{\partial\psi_3}{\partial y}\vec{i}\right) \Rightarrow$$

$\Rightarrow$ uvažimo (*) $\Rightarrow \vec{V}_2 = \frac{1}{2}\left[\left(\frac{\partial\psi_1'}{\partial x} + \frac{\partial\psi_3'}{\partial x}\right)\vec{j} + (U_1 + U_3)\vec{i}\right]$

- dakle, vidimo da u $\vec{V}_2$ nema ovisnosti o y $\Rightarrow \frac{\partial\vec{V}_2}{\partial y} = 0$
- nadalje, vrijedi: $U_m = \frac{U_1 + U_3}{2}$; $\psi_m = \frac{\psi_1' + \psi_3'}{2} = \psi_2'$ $\Rightarrow$

$$\Rightarrow \vec{V}_2 = U_m\vec{i} + \frac{\partial\psi_m}{\partial x}\vec{j} \quad ; \quad U_T = \frac{U_1 - U_3}{2}$$

- sada gledamo sljedeći član:

$$\frac{\partial\vec{V}_2}{\partial x}\cdot\nabla(\psi_1 - \psi_3) = \frac{\partial^2\psi_m}{\partial x^2}\vec{j}\cdot\left[\frac{\partial}{\partial x}(\psi_1 - \psi_3)\vec{i} + \frac{\partial}{\partial y}(\psi_1 - \psi_3)\vec{j}\right] = \frac{\partial^2\psi_m}{\partial x^2}\underbrace{(-U_1 + U_3)}_{-2U_T} =$$

$$= -2U_T\frac{\partial^2\psi_m}{\partial x^2} = -2U_T\frac{\partial^2\psi_2'}{\partial x^2} = -2U_T\xi_2'$$

- pogledajmo $\xi_g = \frac{1}{f_0}\nabla^2\phi = \nabla^2\psi \Rightarrow \xi_2 = \nabla^2\psi_2 = \frac{\partial^2\psi_2}{\partial x^2} = \frac{\partial^2\psi_2'}{\partial x^2} = \xi_2'$

$$\Rightarrow \boxed{\vec{Q}} = -\frac{f_0}{\delta p}\frac{\partial\vec{V}_g}{\partial x}\cdot\nabla(\psi_1 - \psi_3)\vec{i} = -\frac{f_0}{\delta p}\frac{\partial\vec{V}_2}{\partial x}\cdot\nabla(\psi_1 - \psi_3)\vec{i} = \boxed{\frac{2f_0U_T}{\delta p}\xi_2'\vec{i}}$$

- sada još ω-jednadžba iz (**):

$$\sigma\left[\frac{\partial^2}{\partial x^2} - 2\underbrace{\frac{f_0^2}{\sigma(\delta p)^2}}_{\lambda^2}\right]\omega_2' = -2\nabla_p\cdot\vec{Q}$$

$$\Rightarrow \sigma\left(\frac{\partial^2}{\partial x^2} - 2\lambda^2\right)\omega_2' = -2\nabla_p\cdot\left(\frac{2f_0U_T}{\delta p}\xi_2'\vec{i}\right) = -2\frac{\partial}{\partial x}\left(\frac{2f_0U_T}{\delta p}\xi_2'\right)$$

$$\Rightarrow \boxed{\sigma\left(\frac{\partial}{\partial x^2} - 2\lambda^2\right)\omega_2' = -\frac{4f_0U_T}{\delta p}\frac{\partial\xi_2'}{\partial x}}$$

6 Pokažite da u dvoslojnom modelu (2LM) u β-ravnini maksimalni rast baroklinе nestabilnosti očekujemo za identitet:

$$k^2 = 2\lambda^2(\sqrt{2}-1)$$

Rj: disperzijska relacija za 2LM glasi:

$$c = U_m - \frac{\beta(k^2+\lambda^2)}{k^2(k^2+2\lambda^2)} + \sqrt{\frac{\beta^2\lambda^4}{k^4(k^2+2\lambda^2)^2} - \frac{U_T^2(2\lambda^2-k^2)}{k^2+2\lambda^2}} \quad ; \quad \lambda^2 = \frac{f_0^2}{6(\delta p)^2}$$

$\Rightarrow$ β-ravnina $\Rightarrow \beta = 0$:

$$c = U_m + \sqrt{-\frac{U_T^2(2\lambda^2-k^2)}{k^2+2\lambda^2}} = U_m + U_T\sqrt{\frac{k^2-2\lambda^2}{k^2+2\lambda^2}}$$

- prisjetimo se: rješenje za ψ_m pretpostavljamo u obliku:

$$\psi_m = Ae^{ik(x-ct)} \quad ; \quad c \in \mathbb{C} \Rightarrow c = c_{Re} + i\,c_{Im}$$

$$\Rightarrow \psi_m = Ae^{ikx}e^{-ikct} = Ae^{ikx}e^{-ik(c_{Re}+ic_{Im})t} =$$

$$= \underbrace{Ae^{kc_{Im}t}}_{\text{AMPLITUDNI DIO (AD)}}\ \underbrace{e^{ik(x-c_{Re}t)}}_{\text{OSCILATORNI DIO (OD)}}$$

- ako postoji c_{Im} i ako je $c_{Im} > 0 \Rightarrow$ amplituda eksponencijalno raste!
- da bi postojao c_{Im}, izraz pod $\sqrt{\ }$ mora biti < 0:

$$\Rightarrow \frac{k^2-2\lambda^2}{k^2+2\lambda^2} < 0 \Rightarrow k^2 - 2\lambda^2 < 0 \Rightarrow 2\lambda^2 > k^2$$

$$\Rightarrow c = U_m + U_T\sqrt{-\frac{2\lambda^2-k^2}{k^2+2\lambda^2}} = U_m + i\,\underbrace{U_T\sqrt{\frac{2\lambda^2-k^2}{k^2+2\lambda^2}}}_{\sim c_{Im}}$$

$$\Rightarrow (AD) = Ae^{kc_{Im}t} = Ae^{kU_T\sqrt{\frac{2\lambda^2-k^2}{k^2+2\lambda^2}}\,t} = Ae^{\alpha t} \quad ; \quad \alpha = kU_T\sqrt{\frac{2\lambda^2-k^2}{k^2+2\lambda^2}}$$

- maksimalni rast će se dogoditi kada α postigne ekstrem:

$$\Rightarrow \frac{d\alpha}{dk} = 0 \Rightarrow \cancel{U_T}\sqrt{\frac{2\lambda^2-k^2}{k^2+2\lambda^2}} + \cancel{U_T}k\frac{1}{\cancel{2}}\left(\frac{2\lambda^2-k^2}{k^2+2\lambda^2}\right)^{-\frac{1}{2}}\frac{-\cancel{2}k(k^2+2\lambda^2)-(2\lambda^2-k^2)\cancel{2}k}{(k^2+2\lambda^2)^2} = 0$$

$$\Rightarrow \frac{2\lambda^2-k^2}{k^2+2\lambda^2} - k^2\frac{\cancel{k^2}+2\lambda^2+2\lambda^2-\cancel{k^2}}{(k^2+2\lambda^2)^2} = \frac{(2\lambda^2-k^2)(k^2+2\lambda^2)-4k^2\lambda^2}{(k^2+2\lambda^2)^2} = 0 \Rightarrow$$

$\Rightarrow \cancel{2\lambda^2 k^2} + 4\lambda^4 - k^4 - \cancel{2\lambda^2 k^2} - 4k^2\lambda^2 = 0 \Rightarrow$ kvadratna jednadžba po k^2: $k^2 = x \Rightarrow x^2 + 4\lambda^2 x - 4\lambda^4 = 0 \Rightarrow$

$$\Rightarrow x_{1,2} = \frac{-4\lambda^2 \pm \sqrt{16\lambda^4 + 16\lambda^4}}{2} = -2\lambda^2 \pm 2\sqrt{2}\lambda^2 = 2\lambda^2(\pm\sqrt{2}-1)$$

$$\Rightarrow k^2_{1,2} = 2\lambda^2(\pm\sqrt{2}-1)$$

- još treba odrediti koje od gornja 2 rješenja daje minimum:

$$\frac{d\alpha}{dk} = U_T\left[\sqrt{\frac{2\lambda^2-k^2}{k^2+2\lambda^2}} - \frac{k^2}{\sqrt{\frac{2\lambda^2-k^2}{k^2+2\lambda^2}}}\frac{4\lambda^2}{(k^2+2\lambda^2)^2}\right]$$

$\Rightarrow$ $\underline{\overline{DZ}}$ Izračunati $\frac{d^2\alpha}{dk^2}$ i pokazati da se maksimum postiže za $k^2 = 2\lambda^2(\sqrt{2}-1)$