$\boxed{12}$ Izračunajte valni otpor za 2D hidrostatičko strujanje preko zvonolike prepreke iz prethodnog zadatka.

Rj: Valni otpor je preko funkcije perturbacije tlaka zraka i funkcije oblika terena (prepreke) definiran na sljedeći način:

$$D = \int_{-\infty}^{+\infty} p(x, z=0) \frac{dh}{dx}\, dx$$

→ ovdje izostavljamo ' za oznake perturbacija (kao i u prethodnom zadatku)

- dakle, $h(x) = h_0 \frac{a^2}{x^2+a^2}$ imamo, treba nam $p(x,z)$!

- znamo: $\bar{u}\frac{\partial u}{\partial x} + \frac{1}{\rho_0}\frac{\partial p}{\partial x} = 0 \Rightarrow \frac{\partial p}{\partial x} = -\rho_0 \bar{u}\frac{\partial u}{\partial x} \Big/ \int dx$

$$\Rightarrow p(x,z) = -\rho_0 \bar{u}\, u(x,z)$$

→ prethodni zadatak...

- nadalje, $\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \Rightarrow \frac{\partial u}{\partial x} = -\frac{\partial w}{\partial z} = -\frac{\partial}{\partial z}\left(\bar{u}\frac{\partial \eta}{\partial x}\right) \Rightarrow$

$$\Rightarrow \frac{\partial u}{\partial x} = -\frac{\partial}{\partial x}\left(\bar{u}\frac{\partial \eta}{\partial z}\right) \Big/ \int dx \Rightarrow u(x,z) = -\bar{u}\frac{\partial \eta(x,z)}{\partial z} \Rightarrow$$

$$\Rightarrow p(x,z) = \rho_0 \bar{u}^2 \frac{\partial \eta(x,z)}{\partial z}$$

- sada nam treba rješenje za $\eta(x,z)$!
- u prošlom zadatku smo rješavali strujanje u nehidrostatičkom režimu [2sp)] kada je $k \gg l$
- sada treba dobiti fju $\eta(x,z)$ za strujanje u hidrostatičkom režimu [1sp)] kada je $k \ll l$ ⇒ ako već niste, onda to odradite sada za $\boxed{DZ}$!
- rješenje glasi: $\eta(x,z) = h_0 a \frac{a\cos(|l|z) - x\sin(|l|z)}{a^2+x^2}$
- dakle:

$$p(x,z) = -\rho_0 \bar{u}^2 h_0 a \frac{a|l|\sin(|l|z) + x|l|\cos(|l|z)}{a^2+x^2}$$

$\Rightarrow p(x, z=0) = -\rho_0 \bar{u}^2 h_0 a |\ell| \frac{x}{a^2+x^2}$

- nagib: $\frac{dh}{dx} = h_0 a^2 \frac{-2x}{(a^2+x^2)^2} = -2h_0 a^2 \frac{x}{(a^2+x^2)^2}$

- sada: $D = \int_{-\infty}^{+\infty} \left(-\rho_0 \bar{u}^2 h_0 a |\ell| \frac{x}{a^2+x^2}\right)\left[-2h_0 a^2 \frac{x}{(a^2+x^2)^2}\right] dx =$

$$= 2\rho_0 \bar{u}^2 h_0^2 a^3 |\ell| \underbrace{\int_{-\infty}^{\infty} \frac{x^2}{(a^2+x^2)^3} dx}_{\text{opet imamo kompl. integr.}}$$

- ovdje: $f(z) = \frac{z^2}{(z^2+a^2)^3}$

- J. lema: $\lim_{|z|\to\infty} |f(z)| = \lim_{|z|\to\infty} \left|\frac{z^2}{(z^2+a^2)^3}\right| = \lim_{|z|\to\infty} \left|\frac{1}{\left(z^{\frac{4}{3}} + \frac{a^2}{z^{\frac{2}{3}}}\right)^3}\right| = 0$

$\Rightarrow$ zadovoljena u obje poluravnine!

- ovdje fja $f(z)$ ima 2 pola 3. reda $(m=3)$:

$$f(z) = \frac{z^2}{(z-ia)^3(z+ia)^3} \Rightarrow \text{polovi}: z = \pm ia$$

- u gornjoj poluravnini: $\frac{1}{2\pi i}\oint f(z)dz = \frac{1}{2\pi i}\int_{-\infty}^{+\infty} f(x)dx$

- u donjoj —II— : $\frac{1}{2\pi i}\oint f(z)dz = \frac{1}{2\pi i}\int_{+\infty}^{-\infty} f(x)dx$

- gornja: $\int_{-\infty}^{+\infty} f(x)dx = 2\pi i \operatorname{Res}_{ia} f(z)$

- donja: $\int_{-\infty}^{+\infty} f(x)dx = -\int_{+\infty}^{-\infty} f(x)dx = -2\pi i \operatorname{Res}_{-ia} f(z)$

dakle, svejedno je oko kojeg pola integriram...

$$\operatorname{Res}_{ia} f(z) = \lim_{z\to ia} \frac{1}{2!}\frac{d^2}{dz^2}\left[\cancel{(z-ia)^3}\frac{z^2}{\cancel{(z-ia)^3}(z+ia)^3}\right] =$$

$$= \lim_{z\to ia} \frac{1}{2}\frac{d}{dz}\left[\frac{2z(z+ia)^3 - z^2 3(z+ia)^2}{(z+ia)^6}\right] =$$

$$= \lim_{z\to ia} \frac{1}{2}\frac{d}{dz}\left[\frac{2z(z+ia)-3z^2}{(z+ia)^4}\right] = \lim_{z\to ia} \frac{1}{2}\frac{d}{dz}\left[\frac{2zia-z^2}{(z+ia)^4}\right] =$$

$$= \lim_{z\to ia} \frac{1}{2}\frac{(2ia-2z)(z+ia)^4-(2zia-z^2)4(z+ia)^3}{(z+ia)^8} =$$

$$= \lim_{z\to ia} \frac{1}{\cancel{2}}\frac{\cancel{2}(ia-z)(z+ia)-\cancel{4}^{\,2}(2zia-z^2)}{(z+ia)^5} =$$

$$= \lim_{z\to ia} - \frac{a^2+z^2+4zia-2z^2}{(z+ia)^5} = -\frac{\cancel{a^2}-\cancel{a^2}-4a^2+2a^2}{32ia^5} =$$

$$= \frac{\cancel{2a^2}}{\cancel{32}_{16}ia^{\cancel{5}\,3}} = \frac{1}{16ia^3}$$

$$\Rightarrow \int_{-\infty}^{\infty}\frac{x^2}{(a^2+x^2)^3}\,dx = 2\pi \cancel{i}\frac{1}{16\cancel{i}a^3} = \frac{\pi}{8a^3}$$

$$\Rightarrow D = \cancel{2}\rho_0 \bar{u}^{\cancel{2}} h_0^2 \cancel{a^3}\underbrace{|\ell|}_{\frac{N}{\bar{u}}}\frac{\pi}{\cancel{8}_4\cancel{a^3}} \Rightarrow D = \frac{\pi}{4}\rho_0 \bar{u} N h_0^2 \quad \checkmark\checkmark$$